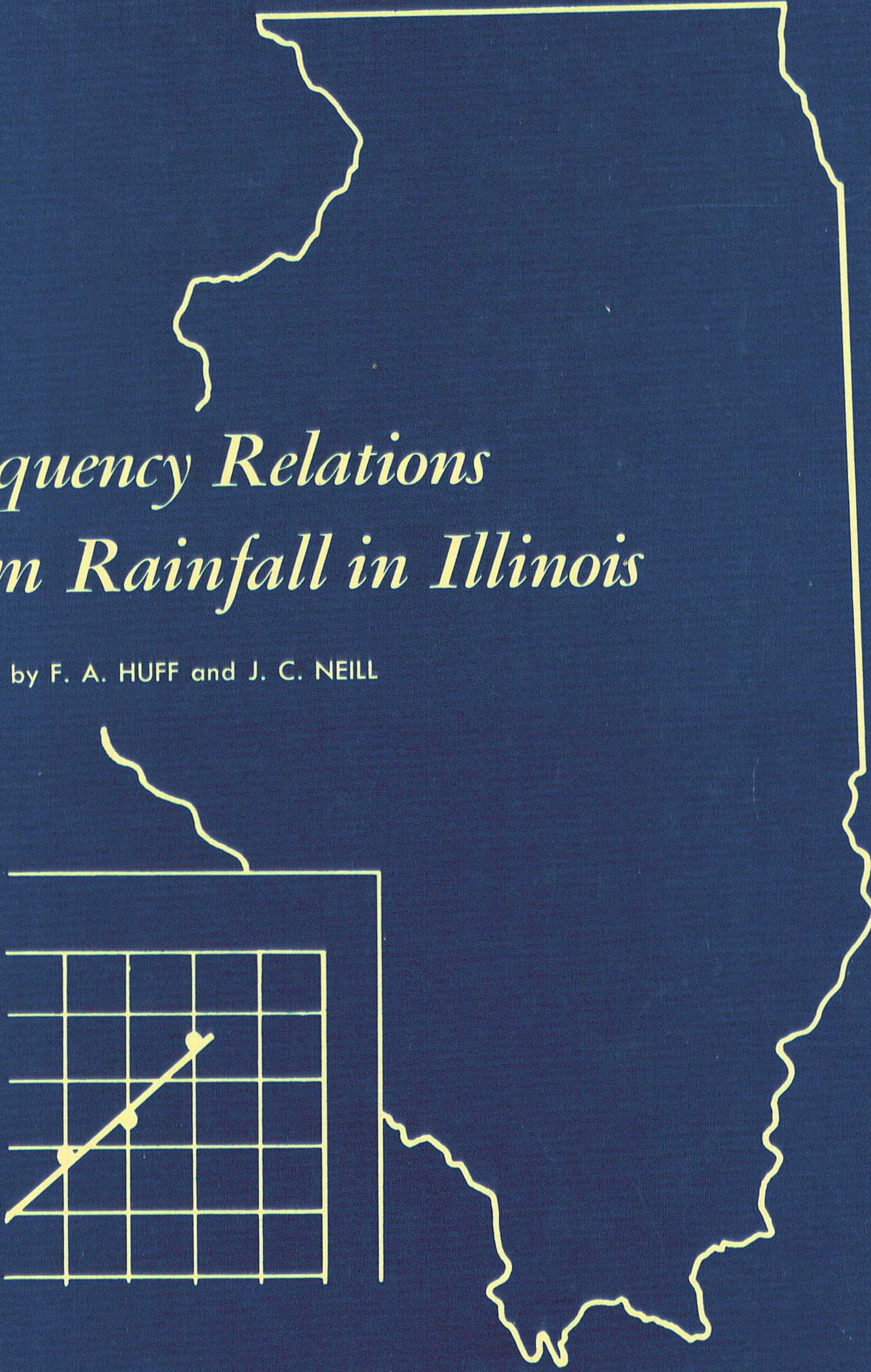
BULLETIN 46

STATE OF ILLINOIS
WILLIAM G. STRATTON, Governor

DEPARTMENT OF REGISTRATION AND EDUCATION
VERA M. BINKS, Director



Frequency Relations for Storm Rainfall in Illinois

by F. A. HUFF and J. C. NEILL

ILLINOIS STATE WATER SURVEY
WILLIAM C. ACKERMANN, Chief

URBANA
1959

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FREQUENCY RELATIONS FOR STORM RAINFALL IN ILLINOIS

by F. A. Huff and J. C. Neill

SUMMARY AND CONCLUSIONS

If the physical laws governing the inception and distribution of precipitation were completely understood, it would be unnecessary to resort to many of the statistical approaches now used by meteorologists and hydrologists to predict the future distribution of storm rainfall. However, even the basic physical processes involved in the production of precipitation in the atmosphere have not been adequately defined or evaluated at present, although considerable research is being directed toward the solution of this problem. Consequently, the investigator of precipitation frequency relations is forced to depend primarily upon application of statistical methodology to samples of observational data which he hopes are representative of the population distribution.

The production and distribution of precipitation are obviously dependent upon complex reactions in nature. Therefore, the authors of this report could find no firm basis for selecting in advance any one of the several commonly used statistical distributions, as best for the analysis of Illinois frequency data. It did not seem logical to pass judgment when so little is known about the basic laws and processes governing rainfall distribution. Rather, it appeared that the selection of a statistical technique should be determined from the goodness-of-fit of raw data to each of several statistical distributions, which appeared as promising yardsticks for estimating future events. Consequently, this approach was followed throughout the investigation in establishing annual and seasonal frequency relations.

Complete objectivity in establishing frequency relations was not possible, since tests of several statistical methods showed that none was distinctly superior in fitting the data samples. Consequently, in the final analysis, the statistical distributions selected for computing annual and seasonal relations were based to some extent upon other available meteorological and climatological information, as discussed in the text. The results represent frequency estimates based upon analysis and evaluation of all information available to the authors during the investigation.

The following conclusions or observations resulted from analyses accomplished in various phases of the investigation of frequency relations.

1. Tests of several statistical distributions commonly used or recommended for frequency analysis of rainfall showed that no one method was distinctly superior in fitting the raw data.
2. Comparison of the methods of moments and least squares in fitting raw data to a given distribution indicated that differences obtained are relatively small and not significant from a practical standpoint.
3. Evaluation of frequency relations developed on both a station and areal basis led to the conclusion that such relations are most reliable when determined for geographical areas. In addition to providing better estimates of average relations, the areal method provides a means for deterraing various probability levels at a point for a given return period. Also, the range of precipitation to be expected within areas of approximately homogeneous precipitation climate can be calculated for given return periods.
4. A comparison was made between frequency relations derived from several statistical distributions and from graphical analysis. Results indicated that simple graphical methods are generally satisfactory for the analysis of frequency data, provided that only a mean frequency curve is required. However, mathematical procedures should be applied when a definition of the reliability of the average relation is needed.
5. Comparison of sectional frequency relations based on data for periods of 10, 20, and 40 years indicated that little difference exists in such relations when the observational period is 20 years or longer.
6. Analysis using data for the partial duration series indicated an excellent fit of areal data when the logarithms of precipitation were related to the logarithms of recurrence intervals.
7. A close association between large-scale and small-scale climatic events was indicated by the magnitude of annual maxima observed during drought and heavy precipitation periods, and by the correlation between annual and storm precipitation.

8. The year-to-year variability of heavy storms is greatest in summer throughout Illinois, with little difference in the time variability in the other three seasons.
9. Analysis of the monthly and seasonal distribution of annual maxima indicated that summer (June-August) is the season of maximum frequency of heavy storms in northern and north central Illinois. In south central Illinois there is little difference among the number of occurrences in summer, spring, and fall; while in the extreme southern part of the state, spring is the season of maximum frequency. As would be expected, winter is the season of minimum frequency throughout the state, but this minimum becomes less pronounced is moving from north to south through the state and differs little from the summer frequency in extreme southern Illinois. The annual maxima occurrences are quite evenly distributed throughout the four seasons in extreme southern Illinois. However, the unusually severe storms which produce the long return period values are most likely to occur in summer over the entire state.

ACKNOWLEDGMENTS

This report was prepared under the general direction of William C. Ackermann, Chief of the Illinois State Water Survey, and the guidance of Glenn E. Stout, Head of the Meteorology Section.

The authors are indebted to Dr. Ven Te Chow, Professor of Hydraulic Engineering, and Dr. H. W. Norton, Professor of Agricultural Statistical Design and Analysis of the University of Illinois, for consultation and suggestions on several aspects of the investigation. The authors are also indebted to Dr. Chow for his review of this manuscript.

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Within the Illinois State Water Survey, credit is due Stanley A. Changnon, Jr., for supervision of the punch card work and drafting of illustrations.

INTRODUCTION

Purpose

Considerable information on precipitation frequencies for periods of 5 minutes to 24 hours has been provided by Yarnell(1) and the U. S. Weather Bureau.(2) However, longer periods of precipitation which are of importance in agriculture, water supply replenishment, and hydrologic design have not been analyzed in detail for Illinois. Consequently, analyses were performed to determine the recurrence interval of heavy precipitation encompassing periods of 1 to 10 days. An early phase of the study was committed to a comparison of several commonly used approaches to frequency analysis of maximum data, in an effort to determine the most representative statistical distribution for use with the Illinois data.

Scope

To accomplish a detailed analysis of heavy precipitation frequencies within the state, annual and seasonal maximum precipitation data were used from 39 stations for the 40-year period 1916-55. Calendar-day data were also utilized in order to include U. S. Weather Bureau cooperative station data,(3) thus providing a relatively large number of stations for determining frequency relations in Illinois. Stations were chosen to give a distribution as uniform as possible throughout the state.

Illinois was divided into four sections of generally similar precipitation climate. For each of these sections, average frequency relations were developed for precipitation periods of 1 to 10 days on both an annual and seasonal basis. In this report, winter includes December through February, and spring encompasses March through May. June through August, and September through November are considered as summer and fall, respectively.

For comparison purposes, an analysis was accomplished on 1-day precipitation data based upon the partial duration series approach. Empirical transformation factors for changing from annual maximum precipitation to partial duration frequencies and from clock-hour maximum to maximum precipitation values were determined for the absolute storm periods of 1 to 10 days. Studies were also made of the actual duration in hours of precipitation during storm periods of 1 to 10 days, the relation between storm and annual precipitation, and the monthly and seasonal distribution of heavy storms.

General Approach

It was decided that the most reliable frequency relations would be obtained by determining a relation to represent the average within an area and then defining the reliability of this average areal relation. Standard errors were used to indicate the range of value's to be expected within each area and thus define the reliability of values chosen from the average curve. Although the study was devoted mainly to determining average frequency relations for sections of Illinois, frequency relations at individual stations were also investigated to ascertain the representativeness of these data for defining frequency relations within the surrounding area.

Obviously, areas chosen for determining average relations should have a homogeneous precipitation climate. However, to obtain areas of sufficient size which would contain enough stations to define reliably the average frequency relation, the authors found it necessary to deal with areas having approximately the same climate.

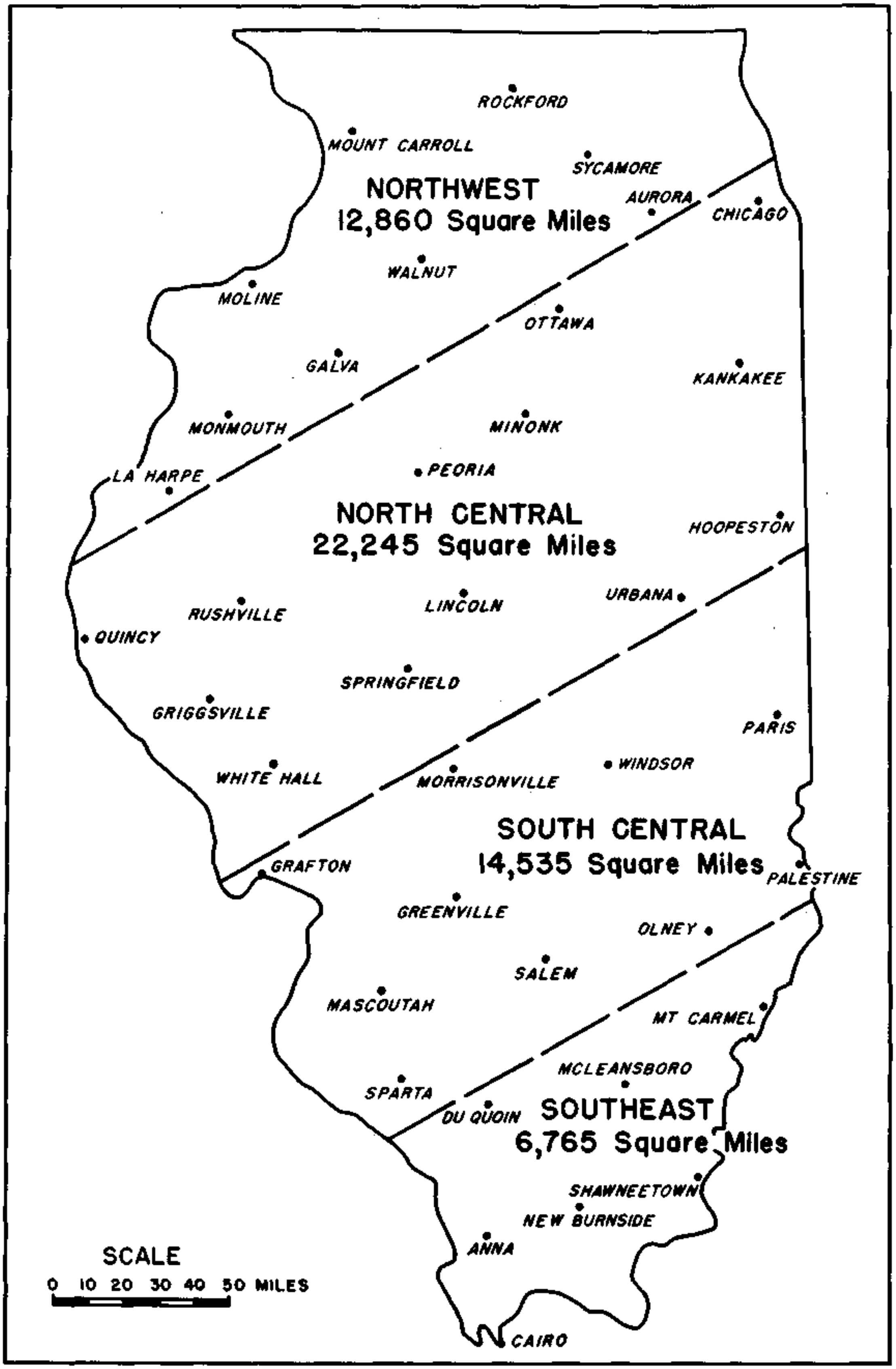

FIGURE 1 SECTION AND STATION LOCATIONS

A number of delineations of sections were tested before the final ones were selected. Station locations, section boundaries and section sizes are shown in Figure 1. The sections were selected on the basis of means, standard deviations, and coefficients of variations of the annual maximum precipitation for the individual stations and on consideration of meteorologic and climatic factors involved. Thus, while the selections were partially subjective, it is believed that the grouping is generally representative of storm rainfall climate within these areas of Illinois. Other groupings tested included a 3-section latitudinal division corresponding to that used by the U. S. Weather Bureau in its state climatological summaries prior to 1957, a 9-section division corresponding to the present climatic sections within the state used by the Weather Bureau, a 3-section longitudinal division, a 4-section latitudinal division, a 4-section division from northwest to southeast throughout the state, and a 4-section division from southwest to northeast.

In addition, an evaluation was made of several statistical approaches to determine if a distinctly preferable analysis technique existed among several commonly used. Frequency curves were fitted to the data using the log-normal distribution and the methods of Gumbel,(4) Jenkinson,(5) Chow,(6) and Frechet.(7) The characteristics of the several methods tested are discussed in another section. For comparative purposes, curves were determined from the raw data by the method of least squares and by the method of moments, except for the Frechet method. In the case of the least squares method of curve fitting, the correlation coefficient and the standard error provide an objective means of comparing the goodness-of-fit for the various frequency distributions.

To help define the range of individual point values about the areal average, standard errors were determined at various recurrence intervals along each regression curve. These standard errors were used in conjunction with corresponding means to determine probabilities covering the range from 10 to *90* percent. Thus, at any given recurrence interval, average rainfall values may be determined from the average curve for the section, along with the expected range of precipitation values for selected probabilities.

Use of average relations for each section creates a discontinuity at the boundary between sections. For example, as one moves from the northern edge of the Southeast Section to the southern edge of the South Central Section (Fig. 1) the frequency relation may abruptly change. The authors feel that this is a minor deficiency which is more than offset by the greater reliability of frequency relations determined by average areal relationships. The average frequency relations could have been used in conjunction with the individual station data to provide isocontour maps for the state for selected recurrence intervals. This may be achieved by using the slope of the areal average curve in conjunction with the mean of the annual maximum precipitation for each station to obtain an adjusted frequency curve for each station. The adjusted frequency curves may then be used to obtain isohyetal maps. The results of this procedure are illustrated in Figure 2, which is a 50-year frequency map for 1-day precipitation obtained by fitting the raw data to the Frechet(7) distribution by the method of least squares. This technique provides a smoothed isohyetal pattern incorporating an area distribution index in the form of the average curve slope. However, the isohyets

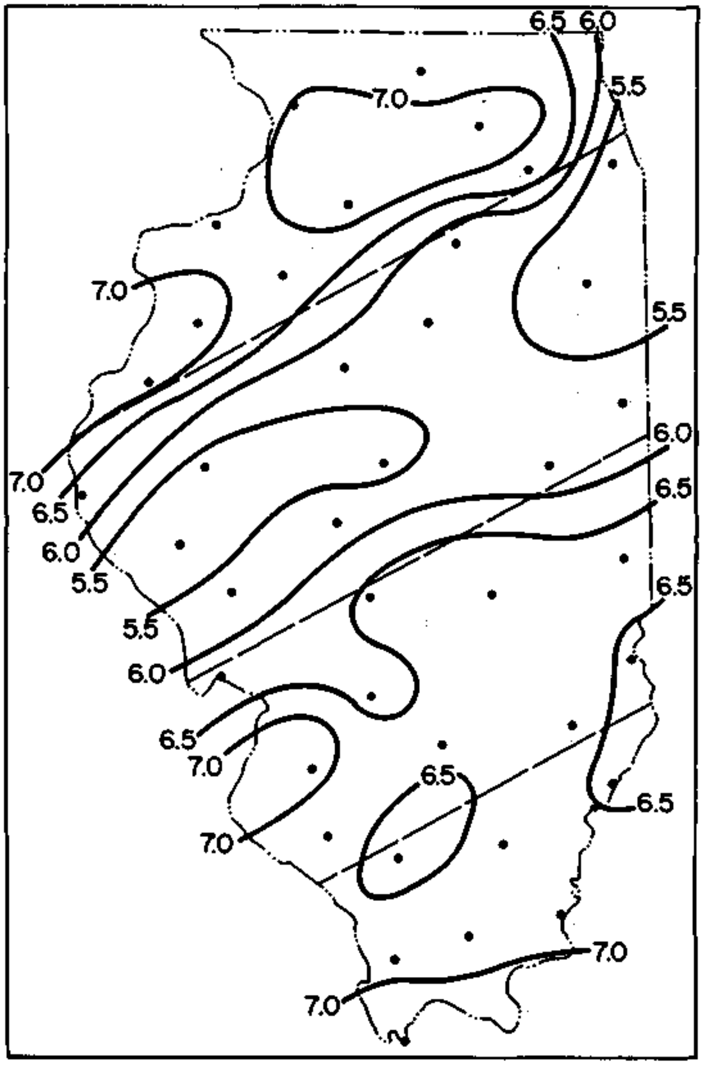

FIGURE 2 50-YEAR RECURRENCE OF 1-DAY PRECIPITATION USING FRECHET METHOD AND DISTRIBUTION INDEX derived from this technique may still be misleading, since even the means of the annual maximum precipitation at stations are considerably affected by extreme values within the sampling period. For comparison, an unsmoothed 50-year frequency map obtained from individual station curves using the Frechet method is presented in Figure 3.

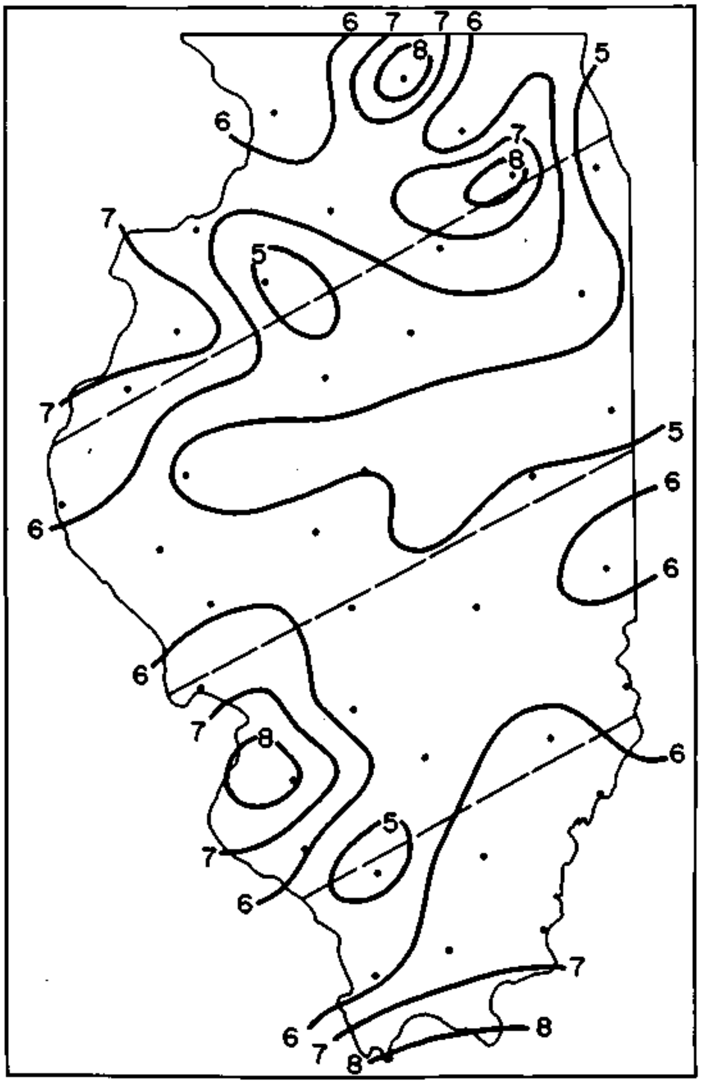

FIGURE 3 50-YEAR RECURRENCE OF 1-DAY PRECIPITATION USING FRECHET METHOD WITHOUT SMOOTHING TECHNIQUE

A simpler cartographic method would assume the section mean values to represent conditions at the central point in each section. An isohyetal map may then be obtained by drawing isohyets parallel to the section boundaries, using linear interpolation between section centers to establish isohyetal values.

As mentioned earlier, the authors prefer to present the study results in the form of areal averages which are believed to provide more realistic values. The sensitivity of isohyetal patterns to sampling variation is illustrated in Figure 4, where station data for the 40-year sampling period have been divided into two groups by the random start sampling technique,[8] and isohyetal patterns drawn for a 50-year recurrence interval of 1-day precipitation with each data group, obtained by fitting the data to the Gumbel distribution.[4] Appreciable differences in the orientation and distribution of isohyets are evident, especially in the northeast and southwest regions of the state.

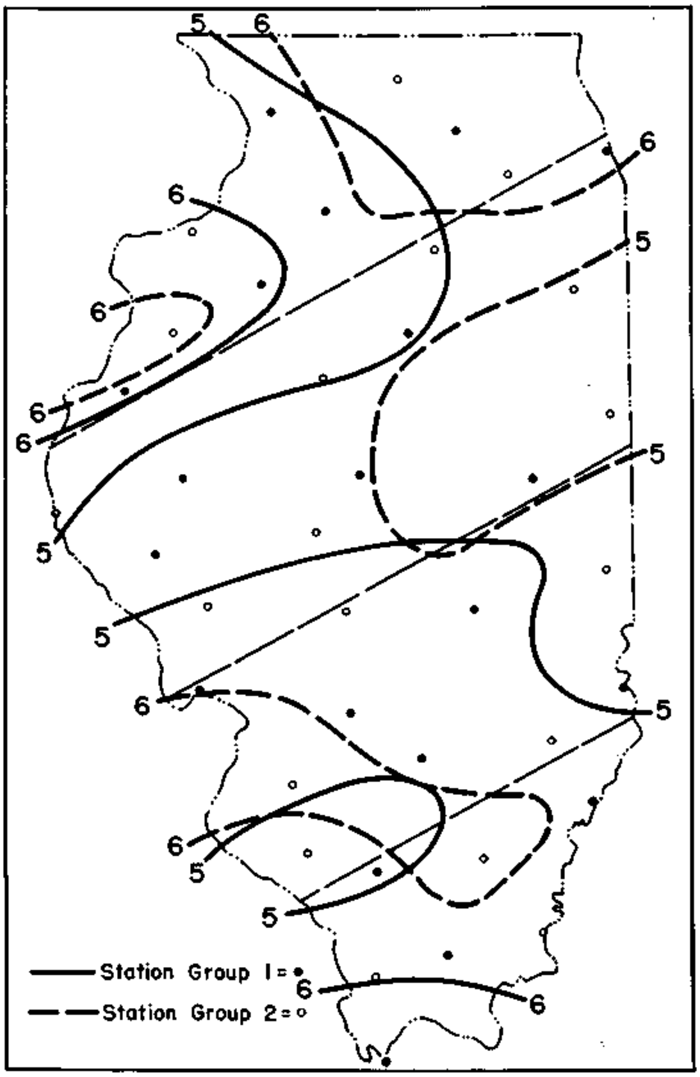

FIGURE 4 COMPARISON OF TWO 40-YEAR ISOHYETAL PATTERNS USING GUMBEL METHOD

Another example to emphasize the desirability of area averages is presented in Table 1, where the range of the maximum 40-year values observed among various stations in each section is shown. Obviously, the location of a station frequency curve is considerably affected by the magnitude of these extreme values, and the relatively large differences which may be obtained between adjacent stations is ap-

TABLE 1

RANGE OF ONE-DAY, 40-YEAR MAXIMUM PRECIPITATION

	Section			
	Northwest	North Central	South Central	Southeast
Range (In.)	4.65- 10.48	3.71- 6.61	4.42- 8.24	4.25- 6.40
Number of Stations	9	13	10	7

parent. For example, the Gumbel distribution fitted to Rockford and Sycamore data in the Northwest Section (Fig. 1) provides 50-year recurrence values of 6.7 inches and 4.7 inches, respectively. If the Frechet method is used, the values become 8.5 inches and 5.2 inches respectively. Analysis of climatological and topographical factors indicates no cause for such marked differences, which are probably due to logical and normal sampling variations. The topography of Illinois is illustrated in Figure 5.(9)

If one assumes that the average 100-year maximum is greater than the. average 40-year maximum at stations, then in a 40-year period such as used in this study, one would expect some of the stations in a representative sample to experience maximum precipitation considerably above their true 40-year average while others should fall appreciably below their true 40-year average. However, if one has chosen a representative sample from an approximately homogeneous area, the average relations developed from the data should provide a reliable estimate of the true frequency relations at points in the selected area. Furthermore, the distribution of the individual points about the mean frequency curve should then provide a measure of the range of values to be expected within the selected area for given recurrence intervals.

Many other examples, similar to that for Rockford and Sycamore, could be presented using data tabulated for this report and data collected by the Illinois State Water Survey on field surveys of severe rainstorms during the past several years. Perhaps, the results of field surveys in recent years have emphasized the desirability of multiple samples and average relations better than any other factors. Some of the implications of these field survey results will be discussed in a later section in conjunction with the selection of the most applicable statistical distribution for use with the Illinois data.

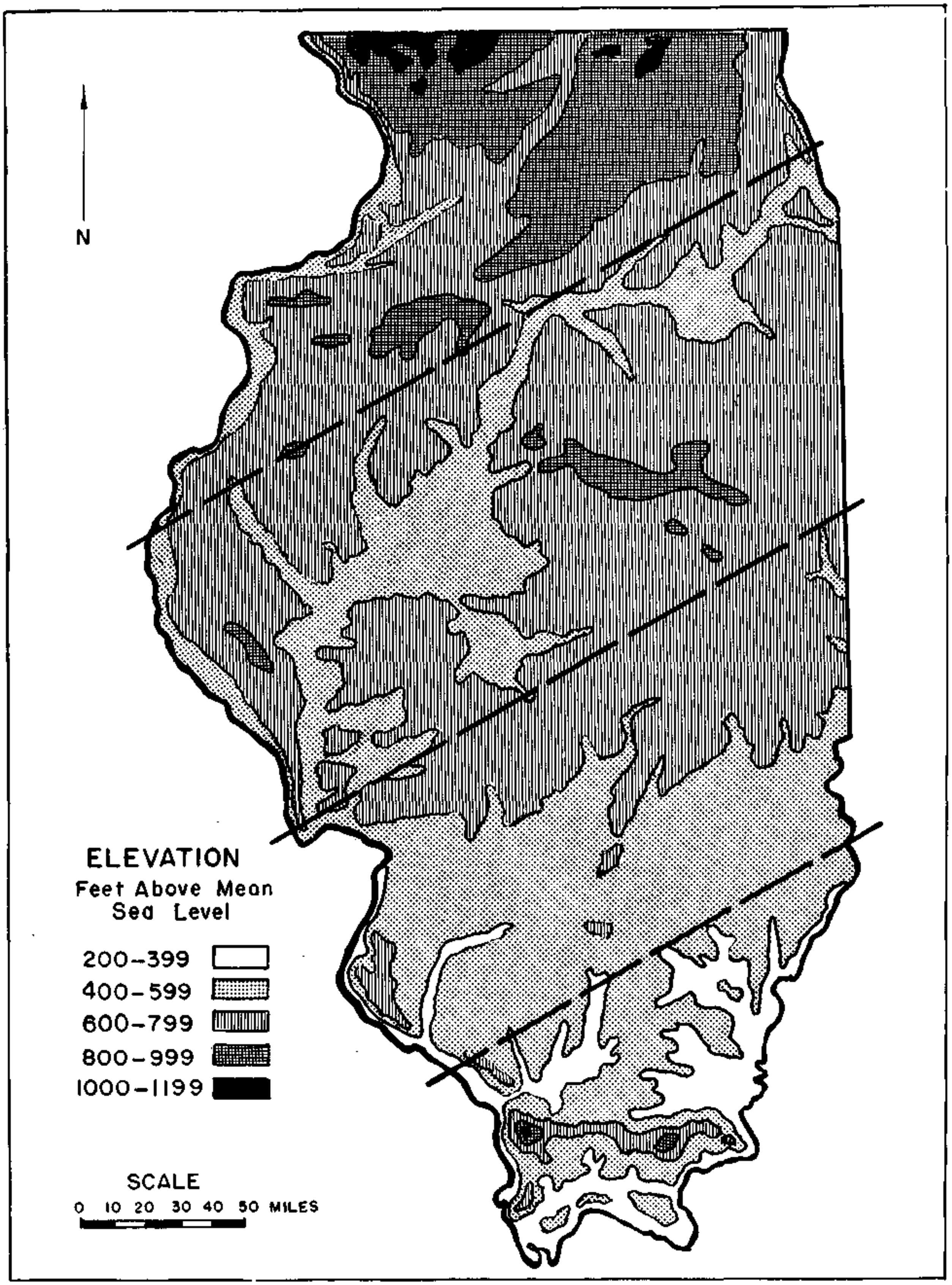

FIGURE 5 ILLINOIS TOPOGRAPHIC MAP

SUMMARY OF FREQUENCY DISTRIBUTION THEORY

Frequency distributions can be classified as either symmetrical or skewed. When a distribution is symmetrical, the mean, mode, and the median coincide. These three parameters are generally separated in the case of skewed or asymmetrical distributions. The amount of separation will depend upon the degree of skewness. Hydrometeorologic variables display various degrees of skewness.

Samples of annual and seasonal maximum rainfall values are expected to exhibit considerable skewness. Skewness is expected even if the maximum values have been drawn from one of the family of normal distributions. This feature of samples of large values may be realized intuitively. When samples are drawn from the upper tail of a unimodal parent population whose upper limit is infinitely large, the probability of getting the larger of the extreme values is less than the chance of getting relatively smaller values. Consequently in repeated sampling, the sample values will tend to be clustered close to the lower end of the large value range. This suggests that large value samples are expected to exhibit various degrees of skewness to the right, i.e., the modal value will be displaced to the left of the median and the median value will be displaced to the left of the mean.

The analysis of extreme value data is concerned with testing the goodness-of-fit of skew distributions and with data transformations which will allow skew data to conform to the characteristics of the normal distribution. In the latter case, goodness-of-fit of the normal distribution to the transformed data would be tested. A brief discussion follows for each distribution that was tested for its ability to describe Illinois maximum annual and seasonal rainfall.

Log-Probability Distribution

Log-Normal. **The normal frequency distribution is defined by**

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(X-m)^2}{2\sigma^2}} \qquad (1)$$

where X is a variate value; m is the population mean; and σ is the standard deviation of the X-variable. The function represents a two parameter family of symmetrical distributions which are completely characterized by m and σ. Changing m will shift the curve to the right or the left without changing the shape. Values for σ measure the degree of concentration of the data about the mean.

The integral or cumulative distribution of Equation (1) will plot as an S-shaped curve when the value of the variable is plotted against cumulative frequency percentage on rectangular coordinate paper. Generally, the same values are plotted on a special graph paper known as probability paper. The cumulative frequency percentage scale is replaced by a modified scale which has die ability to change the cumulative normal distribution into a straight line. The mean value of the distribution plots at the 50 percent point and the slope of the line is proportional to σ.

The normal distribution curve will not adequately describe maximum annual and seasonal rainfall, since diese data are considerably skewed. A transformation of the data is necessary to reduce the skewness factor to a value within the limits of chance variation before the normal distribution function can be applied.

A variable is considered to have a log-normal distribution if the logarithm of the variable can be described by the normal distribution function. The mean and the standard deviation of the logarithm of the variables replace m and σ in plotting the cumulative straight line. The two-parameter log-normal distribution was tested for goodness-of-fit to maximum annual and seasonal rainfall under the assumption that skewness of the logarithms was not statistically significant.

The curve fitting can be done by either the method of moments or by the method of least squares. Curve fitting by the method of moments requires sample estimates $\bar{X}$ and s for m and σ, respectively. These estimates are computed from standard formulas for $\bar{X}$ and s. The line may be constructed on logarithmic probability paper from the location of two points. One point may be located at $X=\bar{X}$ and P = 50 percent, where P is the probability of a value being equal to or less than X. A second point may be located at $X=(\bar{X}-s)$ and P= 15.9 percent. Data points may be plotted around the line for visual inspection of goodness-of-fit. Plotting positions for data points are located at X and P' = M/N+1, where M is the ascending rank of an individual observation, X, N is the total number of observations, and P represents an estimated cumulative percentage.

The P values must be replaced by corresponding standard normal deviates or standard normal frequency factors, before the method of least squares can be used. Standard normal deviates are generally defined mathematically as

$$y = (X - m)/\sigma \qquad (2)$$

The y-deviates are tabulated in any standard normal integral table. The logarithms of the X-variable are determined, and a plot relating log X to y is assumed to linearize the data. The least squares process is then performed on a log X versus y - coordinate system by fitting

$$\log X = \overline{\log X} + b(y - \bar{y}) \qquad (3)$$

to the data, where $\overline{\log X}$ is the mean of the logarithms, b is the slope of the line, and y is the mean of the y-values.

Log-Skew. Chow$^{(6)}$ has proposed a method of logarithmic curve fitting which provides for a variable degree of skewness. The process is essentially a way of adjusting the log-normal probability curve for various skew values.

The adjustment for skewness is based on the log-normal distribution in conjunction with a general formula for hydrologic frequency analysis which Chow expressed as

$$X = \bar{x} + \sigma_x \kappa \qquad (4)$$

where X is an observed value of a variable, x is the mean value of the variable, σ_x is the standard deviation of the variable, and κ is defined as a frequency factor which depends upon the law of occurrence of the variable being analyzed. A mathematical definition of κ may be expressed as

$$\kappa = (X - \bar{x})/\sigma_x \qquad (5)$$

When theoretical expressions for the moments, $\bar{x}$ and σ_x of the log-normal distribution are substituted into Equation (5), the following expression for κ is obtained

$$\kappa = \left[\left(e^{\sigma_z \kappa_z - \sigma_z^2/2}\right) - 1\right] / \sqrt{e^{\sigma_z^2} - 1} \qquad (6)$$

in which

$$\kappa_y = (y - \bar{y})/\sigma_y \qquad (7)$$

where $z = \log_e X$. Equation (7) is therefore analogous to Equation (5) and defines the frequency factor for $\log_e X$ or z which is assumed to be a normally distributed variable. It is now apparent that Equation (6) expresses the frequency factor, *K,* in terms of the frequency factor and standard deviation of a normally distributed variable.

Expressing κ in terms of κ_z and σ_z offers an advantage because values of K_Z are tabulated in the normal probability integral table. Consequently, for any tabled probability level of the normal probability integral a corresponding K_Z or normal deviate can be read.

A value for σ_z is the next requirement for estimating *K* from Equation (6). By allowing the data to suggest a value for the coefficient of variation, C_v, a theoretical value of the coefficient of skewness, C_s, can be computed from the relation

$$C_s = 3C_v + C_v^3 \qquad (8)$$

developed by Chow.$^{(6)}$ Using a C_s value from Equation (8), a theoretical value of σ_z can be obtained from a Chow relationship between σ_z and C_s expressed as

$$\sigma_z = \sqrt{\left[\log_e \sqrt[3]{1 + C_s/2 + (C_s\sqrt{4 + C_s})/2} \;+\; \sqrt{1 + C_s^3/2 - (C_s\sqrt{4 + C_s^2})/2 - 1}\right]} \qquad (9)$$

Theoretical estimates of the original frequency factor, κ, can now be obtained by substituting the single estimate of σ_z into Equation (6) along with as many K_Z values as desired.

Estimated values of the original X-variable can be computed by substituting the *K* estimates in Equation (4) along with estimates of $\bar{x}$ and σ_x which can be computed by using standard formulas. Several values of computed X plotted with corresponding values of P (which are associated with *K)* will outline the fitted line on log-normal probability paper. The fitted line will be curved, its curvature depending upon the size of the skewness factor used in computing κ.

The process of fitting log-skew probability curves to data is done theoretically by the method of moments, but from a practical standpoint, the method of least squares may be used. In using least squares analysis, values of κ for each P' have to be computed as described above. Then the least squares fitting process is performed on a log X versus κ

coordinated system with a linear trend line being assumed. The application of the log-skew probability curves was tested to a limited extent for goodness-of-fit as compared with the log-normal and other frequency distribution.

Gumbel Extreme Value

Fisher and Tippett(10) developed the theory for the distribution of extreme values. Gumbel(4) was the first to apply the theory in the field of hydrology. Consequently, this application of the extreme value law is known as the Gumbel distribution.

Extreme value theory applies to either the largest or the smallest values of N independent sets of values, where each set is composed of M independent values and all are drawn from the same population. The initial population from which the N extremes are drawn must follow a simple exponential law so that it is bounded neither on the upper nor lower ends and the frequency of occurrence must approach zero as the variable increases or decreases.

In practice, extreme value theory has been applied when some of the above conditions were only partially met. For example, it has been applied in cases where either upper or lower limits were bounded, but the boundary limits were considerably beyond the range of observation. The theory assumes that both N amd M are large.

Gumbel introduced a reduced variable

$$y = \frac{X - \mu}{\beta} \qquad (10)$$

into the Fisher and Tippett(10) probability function and expressed the probability, P, of an observation being less than X as

$$P = e^{-e^{-\frac{(X-\mu)}{\beta}}} \qquad (11)$$

The distribution of the maximum value of X is the first derivative of (11) and may be expressed as

$$p = \frac{1}{\beta} e^{-\frac{(X-\mu)}{\beta}} e^{-\frac{(X-\mu)}{\beta}} \qquad (12)$$

Equation (12) represents a family of bell-shaped curves which are similar to the normal distribution curves except they are skewed. The mode, μ, is displaced to the left of the mean. The β factor is equal to $0.78\sigma_x$ and is somewhat analogous to the standard deviation of the normal distribution in that it provides a measure of dispersion of the maximum values about their mode. Gumbel postulates an average skewness factor of 1.139 for the distribution.

A substitution may be used to simplify Equation (12), and to reduce the two-parameter family of curves to a standard distribution curve analogous to the standard normal distribution. The integral of the standard distribution has been tabled so that corresponding values of P and y are readily available.

Gumbel used the method of moments to obtain the following asymptotic relations for

$$\mu = \bar{x} - \beta y \qquad (13)$$

and

$$\beta = \frac{\pi}{\sqrt{6}\, s_x} \qquad (14)$$

where $\bar{X}$ and s_x are the sample estimates of their corresponding population parameters (mean and standard deviation) and J is Eulers' constant which is equal to 0.5772.

Moment estimates of M and β are obtained from substituting sample values, $\bar{X}$ and s_x, in Equations (13) and (14)- Moment estimates are then substituted in Equation (12) to obtain a value of y which is a standard normal deviate having the same probability of being exceeded as any corresponding value of the variable. Probabilities that any y-value and the corresponding value for X will be exceeded may be obtained from a table for the integral of Equation (12)- Values of P and X will plot as a straight line on a special probability paper with X on one axis and a special probability scale for P or y on the other axis. This method of fitting the Gumbel curve to data follows the method of fitting by moments.

The mean value for the extreme value distribution does not plot at the 50 percent point as is the case with the log-probability law. Integrating to the mean values would include nearly 60 percent of the distribution, depending on the degree of skewness of the sample.

Various authors of papers on fitting curves to extreme value data have suggested that the fitting may be done by the method of least squares. When the sample size is small, this may be a more efficient fitting process than the method of moments. The least squares process is done on a y versus X diagram, where the y-values replace their corresponding P values. Essentially, the y-scale is used to linearize the data which would plot as a curved trend on other diagrams.

It is interesting at this point to note that Chow(6) has shown that the Gumbel distribution(4) may be considered as a special case of the log-probability law. The correspondence of the two distributions occurs at the point where the log-probability distribution has a skewness value of 1.139. The variable skewness feature of the log-probability appears to allow for greater flexibility in curve fitting.

Frechet Method

Another form of a two-parameter distribution, which is similar to the Gumbel distribution, was proposed by Frechet and discussed by Gumbel.(7) A logarithmic transformation of the data is necessary before the curve fitting process is done. This is analogous to a transformation from the normal distribution to the log-normal distribution. The cumulative probability may be written

$$P = e^{-e^{-\frac{(\log X - M)}{\alpha}}} \qquad (15)$$

where

$$M = (\Sigma \log X)/N \qquad (16)$$

and

$$\alpha = \pi/\sqrt{6}\, s_{\log X} \qquad (17)$$

where

$$s_{\log X} = \sqrt{\frac{\Sigma(\log X - M)^2}{N - 1}} \qquad (18)$$

which is the standard deviation for the logarithmic values.

The method of fitting the Frechet distribution to a sample of data may be done by either the method of moments or by the method of least squares. The curve fitting processes are analogous to those described for the Gumbel distribution.

Jenkinson Extreme Value

Jenkinson(5) discusses what he considers a defect in the Gumbel extreme value equation. Instead of the special relation between y and X which Gumbel expressed as $y = (X - \mu)/\beta$, Jenkinson suggests a more general relation between y and X which may be written as

$$y = \frac{x - m}{\sigma_1} = \frac{k! - (\log_e [1/(1 - p)])^k}{\pm [(2k)! - (k!)^2]^{1/2}} \qquad (19)$$

where X is an individual maxima, m is the mean of the maxima, σ_1 is the standard deviation of the maxima, P is the probability of the nonexceedance, and k is expressed as follows

$$\sigma_1/\sigma_2 = 2^k \qquad (20)$$

Jenkinson defines the σ_2 factor in Equation (20) as the standard deviation of the greater members in pairs of annual maxima. An estimate of the σ_2 factor may be computed from the data by

$$s_2 = \frac{\Sigma [(X_M - \bar{x})^2 (2M - 1)]}{\Sigma (2M - 1)} \qquad (21)$$

where M is the rank of an observed maxima, X, in ascending order.

The ratio σ_1/σ_2 has the effect of adjusting skewness upward and downward from the average skewness of 1.139 in the Gumbel equation. This feature permits the plotting of concave upward and concave downward curves on Gumbel probability paper in addition to the Gumbel straight line.

Empirical Techniques

Scatter diagrams of the data were plotted on several coordinate systems during the early phases of the analysis. Cube root and square root transformations of maximum rainfall were tested graphically. Semi-logarithmic diagrams were tried with rainfall on the linear scale and recurrence interval on the logarithmic scale. The best linear trend line was obtained when the logarithms of rainfall were related to the logarithms of recurrence intervals. Among these, the log-log plot appeared to describe the data well enough to justify including it in the goodness-of-fit test with the theoretical distributions discussed previously.

Analysis of Statistical Parameters

As part of the analysis in determining frequency relations for Illinois rainfall, computations were made of means, coefficients of variations, skewness coefficients, and Jenkinson(5) shape factors. This was done for storm periods of 1 to 10 days on an annual and seasonal basis for both station and sectional data for 1916-55.

A comparison between means and coefficients of variation on an annual and seasonal basis in the four sections is shown in Table 2. Reference to the means for the annual data show the expected latitudinal trend except for the North Central Section. The increase of precipitation intensity with decreasing latitude is quite evident in the means for winter, where a very significant increase occurs from the Northwest to the Southeast Sections. During the winter, snowfall is frequent in the Northwest Section, while rain accounts for practically all precipitation in the Southeast Section. A latitudinal trend is apparent throughout the four sections in the spring, but not to the extent of the winter trend. In summer, the departure of the North Central Section from the latitudinal trend becomes apparent and maintains itself through the fall season.

Means of annual maxima for 1-day precipitation at individual stations are shown in Figure 6. Similar analyses were performed for other storm periods and for seasons, but have not been reproduced here since they were used primarily for establishing section boundaries within the state.

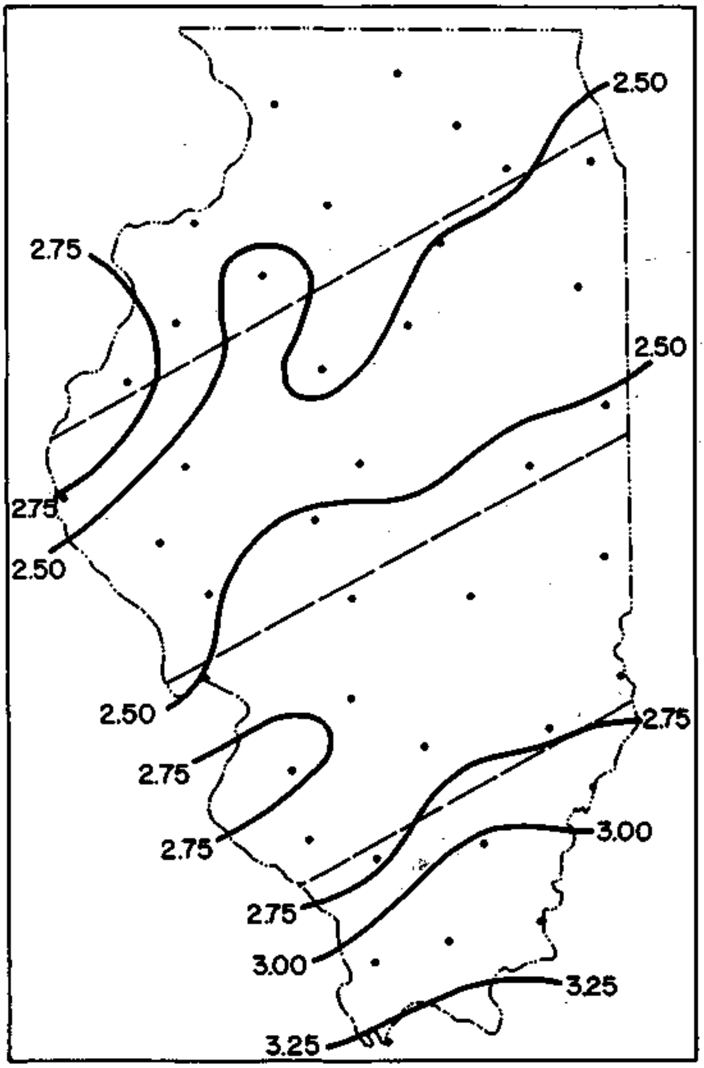

FIGURE 6 MEANS OF ANNUAL MAXIMA FOR 1-DAY PRECIPITATION

TABLE 2

COMPARISONS BETWEEN MEANS AND COEFFICIENTS OF VARIATIONS, 4 SECTIONS, ANNUAL AND SEASONAL, 1916-55

	Annual		Winter		Spring		Summer		Fall	
	Mean	CV	Mean	CV	Mean	CV	Mean	CV	Mean	CV
1-Day Precipitation										
NW	2.62	0.40	1.02	0.42	1.51	0.39	2.16	0.45	1.86	0.49
NC	2.46	0.33	1.09	0.45	1.60	0.41	1.99	0.44	1.68	0.42
SC	2.68	0.35	1.38	0.43	1.72	0.40	2.05	0.46	1.91	0.45
SE	3.01	0.31	1.89	0.48	2.10	0.41	2.05	0.44	2.01	0.44
Avg.		0.36		0.45		0.40		0.45		0.45
2-Day Precipitation										
NW	3.14	0.37	1.22	0.39	1.91	0.40	2.55	0.45	2.21	0.49
NC	2.95	0.32	1.31	0.47	1.97	0.40	2.35	0.42	2.09	0.44
SC	3.31	0.35	1.69	0.49	2.18	0.41	2.45	0.47	2.35	0.46
SE	3.72	0.29	2.34	0.47	2.68	0.43	2.45	0.45	2.48	0.44
Avg.		0.33		0.46		0.41		0.45		0.46
3-Day Precipitation										
NW	3.40	0.35	1.27	0.40	2.10	0.40	2.79	0.44	2.41	0.46
NC	3.27	0.31	1.39	0.49	2.17	0.39	2.57	0.42	2.33	0.45
SC	3.61	0.34	1.84	0.49	2.40	0.40	2.66	0.48	2.53	0.45
SE	4.07	0.28	2.56	0.47	2.92	0.42	2.61	0.45	2.73	0.44
Avg.		0.32		0.46		0.40		0.45		0.45
5-Day Precipitation										
NW	3.85	0.35	1.45	0.39	2.37	0.37	3.14	0.43	2.69	0.49
NC	3.68	0.30	1.57	0.46	2.48	0.38	2.90	0.41	2.60	0.47
SC	4.04	0.33	2.05	0.47	2.82	0.41	2.99	0.46	2.75	0.44
SE	4.49	0.30	2.93	0.47	3.27	0.41	3.00	0.48	2.88	0.43
Avg.		0.32		0.45		0.39		0.45		0.46
10-Day Precipitation										
NW	4.64	0.35	1.69	0.38	3.00	0.35	3.87	0.38	3.26	0.49
NC	4.49	0.30	1.90	0.42	3.16	0.36	3.57	0.39	3.13	0.47
SC	5.05	0.32	2.53	0.49	3.56	0.43	3.74	0.42	3.34	0.44
SE	5.59	0.29	3.66	0.52	4.05	0.38	3.78	0.45	3.58	0.42
Avg.		0.32		0.45		0.38		0.41		0.46
1 to 10-Day Combined Averages										
NW		0.37		0.40		0.38		0.43		0.48
NC		0.31		0.46		0.39		0.42		0.45
SC		0.34		0.47		0.41		0.44		0.45
SE		0.29		0.48		0.41		0.45		0.43
Over-all		0.33		0.45		0.40		0.44		0.45

Reference to Table 2 shows that coefficients of variation (CV) are relatively constant for precipitation periods of 1 to 10 days. As might be expected, the coefficient is somewhat higher for the seasonal than for the annual data. Among the various seasons, the relative variability appears to-be approximately constant except for a slight tendency for a minimum in the spring. Considering both seasonal and annual data, no outstanding trend that one section has greater relative variability than the others is shown. Coefficients of variation of 1-day precipitation for annual maximum precipitation at individual stations are shown in Figure 7.

As an aid in determining the most applicable statistical method of frequency analysis for precipitation periods of 1 to 10 days, skewness coefficients for both individual stations and sectional data were calculated. In addition, the Jenkinson shape factor was calculated. Coefficients of skewness, based upon analysis of station data, are shown in Figure 8 for a 1-day precipitation period. Similar patterns and values were obtained for other storm periods. Results of the sectional analysis are summarized in Table 3, which shows the skewness coefficients (Cs) and shape factors (S_1/S_2) for each section and for annual and seasonal data for 1 to 10 day precipitation periods.

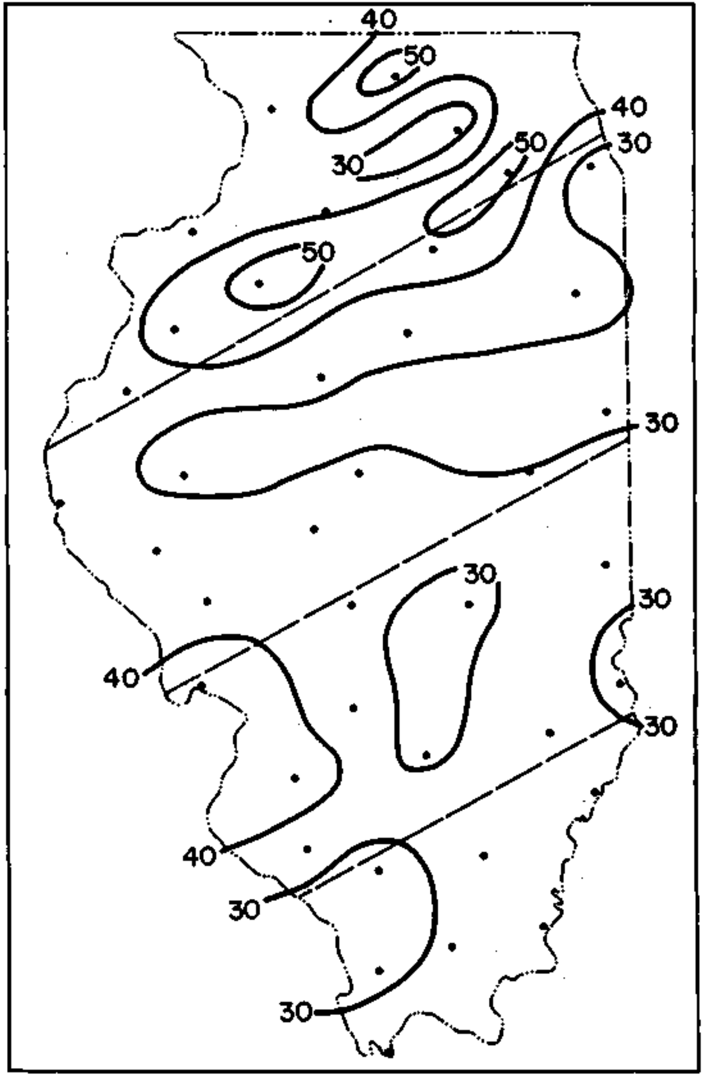

FIGURE 7 COEFFICIENTS OF VARIATION *i%)* OF ANNUAL MAXIMA FOR 1-DAY PRECIPITATION

TABLE 3

COMPARISONS BETWEEN SKEWNESS AND JENKINSON SHAPE FACTOR, 4 SECTIONS, ANNUAL AND SEASONAL

	Annual		Winter		Spring		Summer		Fall	
	Cs	S_1/S_2	Cs	S_1/S_2	Cs	S_1/S_2	Cs	S_1/S_2	Cs	S_1/S_2
					1-Day Precipitation					
NW	1.63	0.84	1.05	0.90	0.71	0.92	1.34	0.86	1.63	0.83
NC	1.14	0.87	1.01	0.88	0.97	0.89	1.08	0.88	1.09	0.88
SC	1.21	0.87	1.02	0.88	0.89	0.91	1.46	0.86	1.14	0.88
SE	0.77	0.91	0.95	0.88	0.71	0.91	1.20	0.87	1.17	0.87
Avg.	1.19	0.87	1.01	0.88	0.81	0.91	1.27	0.87	1.26	0.87
					2-Day Precipitation					
NW	1.43	0.86	1.03	0.89	0.74	0.92	1.37	0.86	1.53	0.86
NC	1.06	0.89	1.16	0.87	0.83	0.91	0.93	0.89	1.10	0.88
SC	1.36	0.86	1.33	0.85	0.75	0.91	1.54	0.86	1.20	0.88
SE	0.61	0.92	0.74	0.90	0.83	0.91	0.98	0.88	0.91	0.89
Avg.	1.12	0.88	1.07	0.88	0.79	0.91	1.21	0.87	1.19	0.88
					3-Day Precipitation					
NW	1.24	0.87	1.13	0.89	0.69	0.92	1.27	0.86	1.30	0.87
NC	0.91	0.90	1.23	0.87	0.74	0.91	0.86	0.90	1.13	0.88
SC	1.48	0.86	1.35	0.86	0.70	0.92	1.52	0.85	1.24	0.88
SE	0.51	0.93	0.90	0.89	0.66	0.92	1.01	0.89	0.98	0.89
Avg.	1.04	0.89	1.15	0.88	0.70	0.92	1.17	0.88	1.16	0.88
					5-Day Precipitation					
NW	1.33	0.86	0.92	0.89	0.52	0.95	1.17	0.88	1.31	0.86
NC	0.93	0.90	0.98	0.89	0.60	0.94	0.81	0.90	1.10	0.88
SC	1.26	0.87	1.41	0.85	0.60	0.93	1.38	0.86	1.21	0.88
SE	0.77	0.91	1.02	0.88	0.71	0.91	1.14	0.87	0.91	0.88
Avg.	1.07	0.89	1.08	0.88	0.61	0.93	1.13	0.88	1.13	0.88
					10-Day Precipitation					
NW	1.21	0.84	1.23	0.87	0.18	0.99	0.92	0.90	1.66	0.85
NC	1.06	0.89	0.94	0.90	0.64	0.93	0.71	0.93	1.41	0.86
SC	0.97	0.89	1.16	0.87	0.80	0.90	1.08	0.89	1.24	0.88
SE	0.80	0.90	1.19	0.86	0.53	0.94	1.05	0.88	0.86	0.90
Avg.	1.01	0.88	1.13	0.88	0.54	0.94	0.94	0.90	1.29	0.87
					1 to 10-Day Combined Averages					
NW	1.37	0.85	1.07	0.88	0.57	0.94	1.21	0.87	1.49	0.85
NC	1.02	0.87	1.06	0.88	0.76	0.91	0.88	0.90	1.21	0.88
SC	1.26	0.87	1.25	0.86	0.75	0.91	1.40	0.86	1.21	0.88
SE	0.69	0.91	0.96	0.88	0.69	0.92	1.06	0.88	0.97	0.89
Over-all	1.09	0.88	1.09	0.88	0.69	0.92	1.14	0.88	1.22	0.88

Reference to Figure 8 shows all stations have positive values of skewness with values ranging from less than one to near four. Maximum values were found in the northern portion of the state. Reference to Table 3 shows a tendency for the highest skewness values in the Northwest Section and the lowest in the Southeast for precipitation values of 1 to 10 days on an annual basis. Considerable fluctuation exists within sections on seasonal basis with a tendency for the South Central Section to have highest skewness values in winter, the South Central and North Central Sections in spring, South Central Section in summer, and the Northwest Section in fall. Seasonally, there is a tendency for slightly higher skewness in fall and a slight trend toward a minimum in the spring or summer. Table 3 also shows a trend for the skewness to decrease with length of precipitation period except for the winter season.

Skewness coefficients for the logarithms of the annual maxima are presented in Table 4. These were computed in order to measure the decrease in skewness and the approach to normality when logarithms are used, such as in the log-normal Frechet, and Chow methods tested in this study. Table 4 shows that logarithmic transformations reduced the skewness considerably, but left the majority of the values positively skewed.

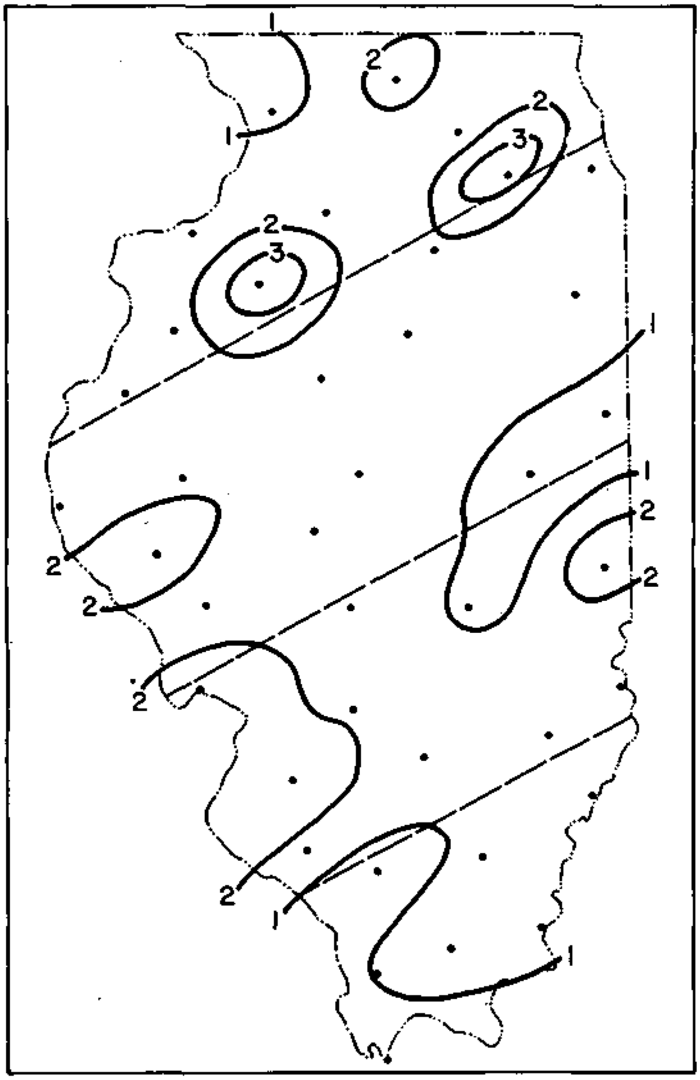

FIGURE 8 SKEWNESS COEFFICIENTS FOR 1-DAY PRECIPITATION

TABLE 4

COEFFICIENTS OF SKEWNESS FOR LOGARITHMS OF ANNUAL MAXIMUM PRECIPITATION

Coefficient for Given Period (Days)

Section	1	2	3	5	10
NW	0.66	0.55	0.32	0.22	0.52
NC	0.50	0.09	0.14	0.24	0.32
SC	0.40	0.36	0.52	0.37	0.07
SE	0.32	-0.19	-0.03	0.25	0.03

Monthly and Seasonal Distribution of Annual Maxima

An investigation was made of the monthly and seasonal distribution of annual maximum precipitation at each of the 39 stations used in the precipitation frequency study. Station data were then combined to obtain the average distribution for each of the four sections. Results of this investigation are summarized in Table 5. The results were instrumental in making the decision to implement the frequency analysis of annual data with a similar analysis of seasonal relations.

In Table 5 the percent of the total number of occurrences of annual maxima occurring in each month and season are given for storm periods of 2, 5, and 10 days for each section. Thus, in the Northwest Section there are 9 stations or a total of 360 recorded annual maxima for each storm period during the 40 years, 1916-55. In this section, the 2-day annual maximum occurred most frequently in June when 18.6 percent or *61* of the 360 values were recorded. Similarly, 45.3 percent of the 2-day values were recorded during the summer season, June through August.

TABLE 5

MONTHLY AND SEASONAL DISTRIBUTION OF ANNUAL MAXIMUM PRECIPITATION

Percent of Total Cases for Given Storm Period (Days) and Section

	Northwest			North Central			South Central			Southeast		
	2	5	10	2	5	10	2	5	10	2	5	10
Jan.	1.4	0.6	1.1	1.4	1.2	1.4	5.8	7.0	9.3	11.1	13.9	13.2
Feb.	0.6	0.6	0.3	0.8	1.0	0.6	2.0	0.8	1.0	3.6	1.1	1.4
Mar.	8.1	7.2	7.2	7.5	5.2	4.3	5.8	6.5	7.0	12.9	11.8	12.5
Apr.	6.7	5.8	7.0	6.4	10.0	9.3	7.5	11.3	11.3	11.1	11.4	11.1
May	5.8	5 3	8.9	10.0	10.0	13.7	14.0	13.3	16.3	8.2	11.1	10.4
June	18.6	22.0	25.8	15.8	19.1	21.2	11.5	12.0	13.0	7.5	7.5	7.1
July	12.5	13.6	11.4	13.9	12.5	11.2	7.5	8.8	7.0	5.4	5.0	7.5
Aug.	14.2	13.6	11.6	12.9	10.2	10.2	10.3	9.3	12.3	9.3	7.1	8.9
Sept.	16.9	16.9	13.9	14.5	14.1	12.7	12.8	11.0	8.0	9.3	11.1	8.2
Oct.	9.4	9.4	9.4	9.5	8.7	8.7	12.3	11.0	7.8	7.5	7.1	7.9
Nov.	3.9	3.9	3.1	4.4	4.8	4.8	7.5	7.3	6.5	7.9	10.0	7.9
Dec.	1.9	1.1	0.3	2.9	2.7	1.9	3.3	2.0	0.8	6.4	2.9	3.9
Dec.-Feb.	3.9	2.3	1.7	5.0	4.8	3.9	11.0	9.8	11.0	21.1	17.9	18.6
Mar.-May	20.6	18.3	23.1	23.9	25.5	27.3	27.3	31.0	34.5	32.1	34.3	33.9
June-Aug.	45.3	49.2	48.8	42.7	41.9	42.6	29.3	30.0	32.3	22.1	19.6	23.6
Sept.-Nov.	30.2	30.2	26.4	28.4	27.6	26.2	32.5	29.3	22.3	24.6	28.2	23.9

Table 5 indicates that the heaviest storms for periods of 1 to 10 days occur most frequently in June in the Northwest and North Central Sections, with a secondary maximum in September. Average monthly rainfall is also highest during June in these two sections. Summer is the season of most frequent heavy storms in both sections. Moving farther south, the South Central Section shows a maximum storm frequency a month earlier in May, which is the month having the the highest mean rainfall in this part of the state. Similar occurrence frequencies prevail for the spring, summer, and fall seasons in this section for storm periods up to five days, while 10-day maxima are most frequent during spring and summer. In the Southeast Section where the winters are relatively mild the variability among months and seasons is considerably less than in the other sections. Here, the month of maximum frequency is March, closely followed by April and January. March and April are the months of highest average monthly precipitation in this region of Illinois. The season of heaviest storms is spring instead of summer as in the more northerly sections.

Next, the annual maxima among the 39 stations were analyzed for, indication of any pronounced trend for increasing or decreasing precipitation intensity during the 40-year period or evidence of any regular cyclic effect in the magnitude of the storms. For this purpose, the upper 10 percent of the annual maxima for each station was used. The station data were then combined to perform a sectional analysis. Results of this analysis are summarized in Table 6, where the distribution of the highest values has been tabulated for storm periods of 2, 5, and 10 days for each of eight 5-year periods comprising the 40 years from 1916 through 1955.

TABLE 6

DISTRIBUTION OF HIGHEST 10 PERCENT OF ANNUAL MAXIMUM PRECIPITATION

Percent of Total Cases for Given Storm Period (Days) and Section

	Northwest			North Central			South Central			Southeast		
Period	2	5	10	2	5	10	2	5	10	2	5	10
1916-20	6	3	3	6	8	4	32	42	25	18	20	4
1921-25	11	8	8	12	13	11	17	10	12	0	4	7
1926-30	22	22	19	19	25	31	3	3	10	7	7	7
1931-35	3	6	0	12	4	4	10	10	3	28	18	14
1936-40	6	11	15	2	4	2	10	5	5	4	11	21
1941-45	11	17	19	21	25	31	10	15	30	14	11	14
1946-50	16	11	17	11	11	9	15	15	15	25	25	33
1951-55	25	22	19	17	10	8	3	0	0	4	4	0

Table 6 does not indicate the presence of any pronounced or regular trend for the precipitation intensity to increase or decrease throughout the 40 years for any of the storm periods. Also no regular cyclic pattern is discernible for these 40 years. Considerable variability of a random or irregular nature does appear, however, and the occurrence frequency appears to correlate well with large-scale climatic events. For example, the frequency of heavy storm periods is relatively low in most cases during the 1930-40 period, which encompassed one of the worst drought periods on record. The 1951-55 frequencies are especially interesting and indicative of the correlation of storm intensity with macroclimatic conditions in the state. During this period, a severe drought was experienced in southern Illinois, while above normal precipitation occurred in the northern part of the state. The correlation is very evident by comparing the frequencies for the South Central and Southeast Sections with those for the Northwest and North Central Sections, in Table 6. Another example of the existing correlation is found in the 1916-20 frequencies. U. S. Weather Bureau records(3) show above normal precipitation for southern Illinois during this period, while the northern portion of the state had considerably below normal precipitation. The correlation between large-scale climatic conditions and the frequency of storm events was also evident in the investigation of the monthly and seasonal frequencies of annual maxima discussed earlier in this section. For example, annual maxima were found to occur most frequently in spring in the Southeast Section. Spring is also the season of heaviest total rainfall in this region.

Frequency Relations for Individual Stations

In the earlier stages of the project, the annual maxima data for individual stations were used to derive frequency relations for each station using the Gumbel, Chow, Jenkinson, log-normal, and Frechet methods. Several other empirical approaches were also tested on a more limited scale by graphical analysis techniques. These included plotting on log-log and semi-logarithmic scales and investigating cube root transformation of the precipitation data.

Results of this phase of the analysis convinced the authors that average sectional relations would be preferable to individual station analysis. Consequently, only a single illustration will be used to indicate the results of this phase. Figure 9 shows the 50-year recurrence values for 1-day annual precipitation, based upon the Gumbel method of analysis. Reference to Figure 9 shows considerable difference between the recurrence values for individual stations in several areas of the state, although there appear to be no climatological or topographical features (Fig. 5) to support these appreciable differences. For example, in the Northwest Section amounts vary from 4.75 to 6.96 inches within a distance of 25 miles from Sycamore to Aurora. In the South Central Section, a value of 6.64 is obtained at Mascoutah while less than 50 miles to the east-northeast a 5.06-inch value is found at Salem. Farther south, values range from 4.66 inches at DuQuoin to 6.25 inches at McLeansboro, about 30 miles to the east. Available evidence indicates that these relatively large dif-

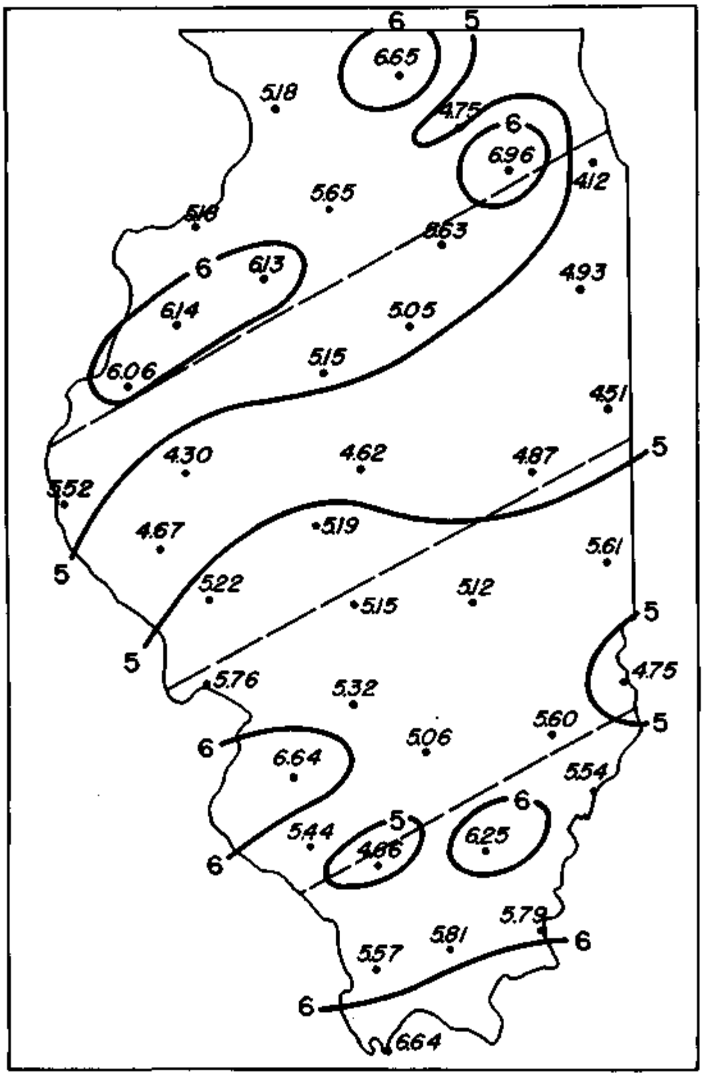

FIGURE 9 50-YEAR RECURRENCE OF 1-DAY PRECIPITATION BASED ON GUMBEL METHOD

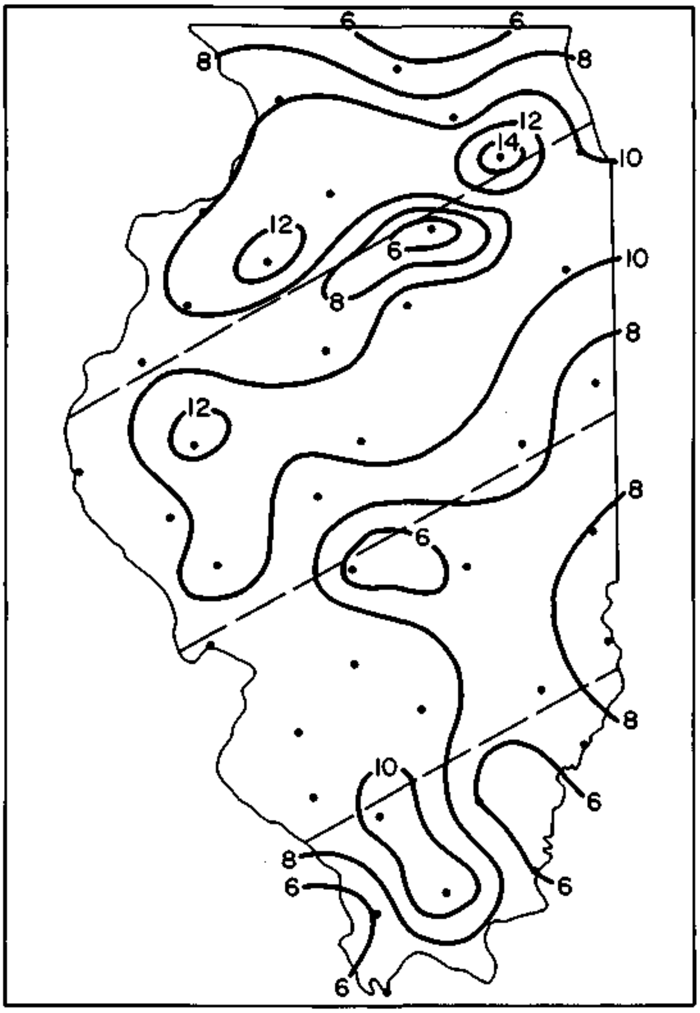

FIGURE 10 MAXIMUM DIFFERENCE (%) BETWEEN 50-YEAR RECURRENCE VALUES OF T-DAY PRECIPITATION USING FOUR ANALYSIS METHODS

ferences are mainly the result of random spatial fluctuations which may be expected within a given sampling period, rather than any real climatic differences. Therefore, based on available evidence it appears that sectional analysis, in which all data are grouped and a measure of the normal variability range provided, is preferable to an isohyetal map presentation utilizing individual station data.

As a part of the evaluation of various statistical methods applied to frequency analysis, the Gumbel, log-normal, Chow, and Jenkinson methods were applied to individual station data. The raw data were fitted to each distribution by the method of moments. The maximum differences obtained between the four methods for a 50-year return period of 1-day annual precipitation is shown , in Figure 10. In this figure, the maximum difference between the methods for each station has been expressed in percent, and calculated by dividing the difference between the highest and lowest return period values by the lowest value. The maximum differences varied from 5 to 15 percent with an average of 9 percent. These maximum differences appear to be quite small, when one considers all the possible sampling variations involved in frequency analysis of station data.

Results of the foregoing analysis indicate little choice between the four statistical methods tested. As discussed earlier in this report, the Chow and Jenkinson methods are more complex statistical techniques. Considering the extra work involved in computations and the small differences obtained in derived frequency relations between these and the simpler methods involving only the first two moments, use of the Chow and Jenkinson methods did not appear desirable in the present study. Although little difference was found in values predicted by the four methods discussed above, the Frechet method gives values which differ appreciably from these four when the recurrence interval reaches 50 years. These differences will be discussed in detail in a later section.

Comparison Between Two 20-Year Periods

During the early phases of the frequency study, considerable effort was given to finding the most applicable statistical method of frequency analysis. As a part of this phase, a comparison was made between the observed rainfall distribution during 1936-55 and recurrence interval values predicted by various statistical methods utilizing 1916-35 data. The frequency distribution of 1-day, 5-day, and 10-day precipitation amounts were calculated from the 1916-35 data using the Gumbel, log-normal, Chow, Jenkinson, and Frechet methods. In addition, the 1916-35 ranked data were assumed to represent the frequency distribution of future events; that is, the raw data were ranked and the ranked values assumed to represent recurrence interval values. Thus, the highest rainfall value observed in the 1916-35 period was assumed to represent the 20-year recurrence value. The Gurabel, log-normal, Chow, and Jenkinson values were obtained by the method of moments following standard procedures. The Frechet distribution was obtained by graphical analysis.

Data for the second 20-year period, 1936-55, were then ranked. These ranked data were next compared with the appropriate recurrence interval values obtained from the 1916-35 data. The analysis was based on average relationships in each of the four climatic sections.

TABLE 7

AVERAGE 4-SECTION ERRORS (%) IN USING VARIOUS STATISTICAL METHODS APPLIED TO 1916-35 DATA FOR ESTIMATING 1936-55 FREQUENCIES

Method	Error (%) for Given Return Period (Yrs.)				
	20	10	5	2	Over-all
		One-Day Amounts			
Ranked Data	5	7	6	4	6
Frechet	5	6	6	4	5
Gumbel	10	6	5	3	6
Log-Normal	18	13	6	4	10
Chow	17	12	6	3	9
Jenkinson	17	12	9	12	12
		Five-Day Amounts			
Ranked Data	9	4	4	5	5
Frechet	7	4	4	4	5
Gumbel	13	4	3	4	6
Log-Normal	20	10	5	4	10
Chow	21	15	5	5	12
Jenkinson	20	12	9	10	13
		Ten-Day Amounts			
Ranked Data	9	9	6	7	8
Frechet	8	5	7	6	6
Gumbel	13	5	6	6	7
Log-Normal	21	8	6	6	10
Chow	22	10	7	6	11
Jenkinson	19	13	9	8	12

Results of this analysis are summarized in Table 7. In this table, an average error is given in percent for the various statistical methods when applied to the 1916-35 data to estimate 1936-55 frequencies. Reference to Table 7 indicates that the best estimates of the 1936-55 precipitation frequencies were obtained from the ranking of raw data, from the Frechet curve, and from the Gumbel method. Results indicate that little is to be gained by a complex statistical treatment of frequency data. The authors realize, of course, that the results are based only upon two 20-year periods and consideration of additional periods might significantly alter the results.

Comparison of Frequency Relations Based on Periods of 10, 20, and *40* Years

A comparison was made of frequency relations obtained from 10-year and 20-year periods with those obtained from the 40-year period, 1916-55. The 1916-25 and 1916-35 data were used for the two shorter periods. This phase of the study was undertaken to obtain a measure of the effect of sampling period in determining frequency relations. For comparison purposes, frequency relations were determined using the Gumbel method applied to average data for each of the four sections and for 10 individual stations distributed throughout the four sections. The station comparison was accomplished for the 20-year and 40-year periods only. Results of the investigation are presented in Tables 8 to 10.

TABLE 8

COMPARISON BETWEEN SECTION FREQUENCY RELATIONS
DERIVED FROM 20-YEAR AND 40-YEAR DATA USING GUMBEL METHOD

	Difference (%) for Given Recurrence Interval (Yrs.)					
Section	40	20	10	5	2	Absolute Average
			1-Day			
Northwest	+5	+4	+4	+5	+5	5
North Central	0	-1	0	0	+1	0
South Central	-2	-3	-2	-3	-3	3
Southeast	-4	-4	-4	-4	-1	3
			5-Day			
Northwest	+2	+2	+3	+3	+3	3
North Central	-1	-1	0	0	+1	1
South Central	-2	-3	-2	-3	-3	3
Southeast	+1	0	0	-1	-1	1
			10-Day			
Northwest	+13	+12	+12	+10	+8	11
North Central	0	- 1	+ 1	+ 1	+3	2
South Central	- 1	- 2	- 2	- 2	-4	2
Southeast	+ 5	+ 4	+ 3	+ 2	-2	3

TABLE 9

COMPARISON BETWEEN SECTION FREQUENCY RELATIONS DERIVED FROM 10-YEAR AND 40-YEAR DATA USING GUMBEL METHOD

	Difference (%) For Given Recurrence Interval (Yrs.)					
Section	**40**	**20**	**10**	**5**	**2**	**Absolute Average**
			1-Day			
Northwest	+4	+4	+6	+7	+11	6
North Central	-3	-2	-1	+1	+ 4	2
South Central	-9	-10	-9	-10	-12	10
Southeast	-4	-4	-2	-1	+ 3	3
			5-Day			
Northwest	+1	+1	+2	+2	+ 5	2
North Central	-4	-3	-2	+2	+ 3	3
South Central	-17	-17	-15	-13	-12	15
Southeast	-4	-3	-3	-3	0	3
			10-Day			
Northwest	+13	+12	+12	+11	+ 8	11
North Central	+ 4	+ 4	+ 4	+ 4	+ 3	4
South Central	-15	-15	-14	-13	-10	13
Southeast	+ 6	+ 6	+ 6	+ 7	+ 7	6

TABLE 10

COMPARISON BETWEEN STATION FREQUENCY RELATIONS DERIVED FROM 20-YEAR AND 40-YEAR DATA USING GUMBEL METHOD

	Difference (%) for Given Recurrence Interval (Yrs.)					
Station	**40**	**20**	**10**	**5**	**2**	**Absolute Average**
Moline	+ 7	+ 7	+ 7	+ 8	+ 8	7
Aurora	+34	+32	+27	+22	+ 6	24
Quincy	+13	+13	+12	+11	+ 5	11
Peoria	- 1	0	+ 2	+ 4	+ 9	3
Hoopeston	-14	-13	-11	-10	- 5	11
Urbana	- 6	- 5	- 4	- 4	- 2	4
Grafton	+33	+31	+28	+21	+ 7	24
Windsor	-18	-16	-16	-16	-17	17
Sparta	-14	-13	-11	- 9	- 3	10
New Burnside	- 6	- 6	- 4	- 3	0	4
Mt. Carmel	- 1	- 1	+ 1	+ 3	+ 6	2

In Tables 8 and 9, a comparison of recurrence interval values is given for each of the four sections for 1-day, 5-day, and 10-day precipitation amounts for selected recurrence periods. Percentage differences were found by subtracting the shorter period values from the 40-year values and dividing by the 40-year values. Reference to Table 8 shows only relatively small differences between the 20-year and 40-year data. Only for 10-day amounts in the Northwest Section did the differences exceed five percent.

As would be expected, frequencies derived from the 10-year and 40-year data do not agree as closely as the 20-year and 40-year results. Table 9 shows excellent agreement between the derived frequencies for the North Central and Southeast Sections, and for 1-day and 5-day precipitation periods in the Northwest Section. However, differences are relatively large for the South Central Section. R.esults of the section analysis indicate that calculated frequency relations become relatively stable when the sampling period is 20 years or greater, and when multiple sampling points are used to define average conditions within an area.

Table 10 shows a percentage comparison between 20-year and 40-year values for 1-day precipitation amounts for 10 selected stations. The results show considerable variability among the stations. For example, only minor differences exist between the recurrence values obtained from 20-year and 40-year data at Mt. Carmel, while at Aurora and Grafton the differences are relatively large. Similar variability among stations was found with other precipitation periods. Comparison of Table 10 with 1-day values of Table 8 further emphasizes the desirability of obtaining section relations based on multiple samples.

Comparison Between Methods of Moments and Least Squares

Perusal of the literature and discussions with prominent statisticians indicate that considerable disagreement exists concerning the best technique of fitting observational data to a selected frequency distribution, such as Gumbel's or Chow's. Some favor the fitting of the data to the chosen frequency distribution by the method of least squares. Others favor use of the method of moments. The question arises as to which is more meaningful, the least-squares fit or one based on the mean and a slope factor determined by a statistic of dispersion about the mean, which is the standard deviation in the case of the normal and log-normal distributions and 0.78 standard deviation in the Gumbel method.

Both the least-squares method and the moments method place the line through the sample mean. However, the method of determining the slope is different. The method of least squares minimizes the sum of squares of deviations about a linear trend line in obtaining the slope. In using the method of moments, the slope is determined by minimizing the sum of squares of deviations about the sample mean value.

Climatological events appear to be governed by natural laws which are very complex and, consequently, it is extremely doubtful that they can be completely defined by a theoretical frequency distribution. This is especially true with precipitation, which is subject to not one, but several physical laws. Statistical analysis methods can provide only a yardstick to help estimate the behavior of natural phenomena. Therefore, it does not seem reasonable to say that one method has a better theoretical basis than the other. It would appear best to test various methods with available data and utilize a least-squares fit with the statistical distribution which appears to be the best yardstick for the existing frequency analysis.

Although the authors favored the least-squares approach to the method of moments, both methods were tested to determine any significant differences between them. Sectional data for the period 1916-1955 were fitted using both techniques with annual and seasonal data. Comparisons were made using the Gumbel, log-normal, Chow and Jenkinson methods. Results of the comparisons are shown for a 1-day precipitation period using annual data for each of the four climatic sections in Table 11. Results of a similar nature were obtained with data for longer precipitation periods and for seasons.

TABLE 11

PERCENT DIFFERENCE BETWEEN METHODS OF LEAST SQUARES AND MOMENTS

1-Day Annual Maximum Precipitation

Recurrence (Yrs.)	Gumbel NW	Gumbel NC	Gumbel SC	Gumbel SE	Log-Normal NW	Log-Normal NC	Log-Normal SC	Log-Normal SE
2	0	0	0	0	0	0	0	0
5	- 1	- 2	- 2	0	+ 1	0	0	+ 2
10	- 2	- 1	- 2	0	- 1	+ 1	0	+ 3
20	- 1	- 1	- 1	+ 1	- 2	0	0	+ 3
40	- 2	- 2	- 2	0	- 2	0	0	+ 4
100	- 2	- 2	- 2	0	- 3	0	0	+ 4
Absolute Avg.	1	1	2	0	2	0	0	3

Recurrence (Yrs.)	Chow NW	Chow NC	Chow SC	Chow SE	Jenkinson NW	Jenkinson NC	Jenkinson SC	Jenkinson SE
2	- 8	- 4	- 5	- 4	+ 3	+ 5	+ 5	+ 5
5	- 9	- 2	- 2	0	+ 5	+ 4	+ 4	+ 3
10	+ 1	+ 2	+ 5	+ 4	+ 8	+ 6	+ 6	+ 5
20	+10	+ 9	+ 9	+11	+12	+ 8	+ 8	+ 6
40	+21	+15	+17	+15	+15	+11	+11	+ 9
100	+34	+26	+21	+23	+17	+12	+13	+ 9
Absolute Avg.	14	10	10	10	10	8	8	6

The percentages in Table 11 were obtained by subtracting moment values from least-squares values and dividing by the least-squares value. This table shows only very small differences between the two methods when applied to Gumbel and log-normal distributions. Differences became appreciable with the Chow and Jenkinson systems, especially at the longer recurrence intervals. Cross-over from negative to positive differences can be noted with the Chow method between the 5-year and 10-year recurrences for all sections and the strictly positive values for Jenkinson throughout. Reference to graphical plots indicated that the Chow and Jenkinson values obtained from the least-squares curves fit the raw data better than those obtained with the moments method, especially above 2-year recurrences which is the region of principal interest to hydrologists.

Comparison of Statistical Distribution Applied to Sectional Annual Maxima Data

In the final evaluation of the several statistical distributions, the method of least squares was applied to sectional data. Use of IBM and digital computer facilities at the University of Illinois greatly facilitated the computations and provided a ready means for the determination of correlation coefficients, standard errors, and coefficients of variation. These parameters were desirable in completing the statistical evaluations prior to final derivation of frequency relations for Illinois rainfall. Least squares regression equations were determined from pooled ranked data for each precipitation period and for annual and seasonal data using the Gumbel, log-normal, and Frechet methods, and an empirical technique in which the logarithms of precipitation are related to the logarithms of the recurrence interval. The latter method was found applicable in fitting partial duration data which will be discussed in a later section and is referred to hereafter as the log-log method. Least-squares regressions were determined for only 1-day precipitation with the Chow (log-skew) and Jenkinson distributions. Previous tests utilizing the method of moments with both station and sectional data indicated that little, if any, improvement could be expected by use of these more complex methods in place of the log-normal and Gumbel methods.

Initially, regression equations were determined for each precipitation period in each section on an annual and seasonal basis. That is, using annual data for each section there would be regression equations corresponding to precipitation periods of 1, 2, 3, 5, and 10 days for each statistical method tested.

Regression equations were obtained utilizing the method of Chow,(11) whereby the data are fitted to a general linear equation:

$$X = \bar{x} + \sigma_x K$$

Where X is precipitation depth (in.), K is a frequency factor which varies with the distribution in use (Gumbel, log-normal, etc.), x is the mean value and σ_x is the standard deviation of the variable. The factor, K, is related to the recurrence interval. Convenient tables for conversion with their methods have been prepared by Chow,(6) and Jenkinson,(5) while Weiss(12) has prepared similar tables for Gumbel. The log-normal factors can be obtained from various statistical texts containing normal probability tables.

Inspection of individual equations for 1, 2, 3, 5, and 10 days of annual data for each section indicated that the slopes of the curves within each section were approximately parallel, except for those derived from the Gumbel method which showed a distinct trend for increasing slope with increasing precipitation period. Consequently, single regression equations were determined for the log-normal, Frechet and log-log methods by adding a parameter for precipitation duration. Graphical analysis of data for storm periods of 1 to 10 days indicated that precipitation for a given recurrence interval varied logarithmically with the precipitation period. Therefore, data were combined into one equation of the form:

$$X = ak + b \log T + c$$

Where X is precipitation, K is a frequency factor as defined by Chow, T is duration of precipitation period in days, and a, b, and c are regression constants.

TABLE 12

CORRELATION COEFFICIENTS FOR REGRESSION EQUATIONS

Period (Days)	Correlation Coefficients for Given Section and Method			
	Northwest	North Central	South Central	Southeast
		Gumbel		
1	0.91	0.95	0.95	0.96
2	0.91	0.96	0.91	0.97
3	0.91	0.96	0.90	0.96
5	0.93	0.96	0.91	0.96
10	0.94	0.95	0.95	0.96
Avg.	0.92	0.96	0.92	0.96
		Log-Normal		
1	0.95	0.96	0.97	0.97
2	0.95	0.96	0.95	0.97
3	0.95	0.97	0.95	0.97
5	0.95	0.96	0.95	0.96
10	0.96	0.96	0.97	0.96
Avg.	0.95	0.96	0.96	0.97
		Frechet		
1	0.96	0.96	0.97	0.96
2	0.96	0.96	0.95	0.96
3	0.95	0.96	0.95	0.95
5	0.95	0.95	0.95	0.96
10	0.96	0.95	0.96	0.95
Avg.	0.96	0.96	0.96	0.96
		Log-Log		
1	0.93	0.93	0.94	0.92
2	0.92	0.93	0.92	0.91
3	0.92	0.92	0.92	0.90
5	0.92	0.91	0.91	0.92
10	0.93	0.92	0.91	0.91
Avg.	0.92	0.92	0.92	0.91
		Chow		
1	0.95	0.96	0.97	0.96
		Jenkinson		
1	0.93	0.96	0.96	0.96

As mentioned previously, several statistical parameters were used to evaluate the statistical distribution methods. Correlation coefficients are presented for individual equations for each period and section using annual data in Table 12. As indicated earlier, calculations for the Chow and Jenkinson distributions were made for only a 1-day precipitation period, because of their complexity which greatly increased the work involved in preparation of the data for analysis with the digital computer. Except for the log-log method, the coefficients in Table 12 show only small differences among the equations derived from the several statistical techniques. Similar results were obtained from comparison of standard errors and coefficients of variation. The analysis to this point established no marked superiority among the several methods in fitting the observational data.

TABLE 13

PERCENT DIFFERENCE BETWEEN RANKED DATA AND REGRESSION CURVE VALUES OBTAINED FROM SEVERAL STATISTICAL METHODS FOR 1-DAY FREQUENCIES

Section	Gumbel	Frechet	Log-Normal	Chow	Jenkinson	Log-Log
			40-Yr. Recurrence			
Northwest	-19	-6	-23	0	-9	+6
North Central	-8	+5	-12	+3	+1	+16
South Central	-16	-3	-19	-4	-8	+7
Southeast	0	+12	-5	-6	+3	+22
Absolute Avg.	11	7	15	3	5	13
			20-Yr. Recurrence			
Northwest	+2	+9	-4	+10	+6	+15
North Central	-7	0	-11	-2	-4	+4
South Central	+1	+8	-3	+7	+4	+12
Southeast	+8	+16	+5	+14	+10	+21
Absolute Avg.	5	8	6	8	6	13
			10-Yr. Recurrence			
Northwest	+2	+1	-2	0	0	+2
North Central	+1	+1	-1	0	-1	+2
South Central	+3	+3	0	+2	+1	+3
Southeast	+5	+6	+3	+4	+3	+9
Absolute Avg.	3	3	2	2	1	4
			5-Yr. Recurrence			
Northwest	+1	-5	-1	-8	-5	-8
North Central	+2	-1	+1	-1	-1	-4
South Central	+1	-2	+1	-2	-2	+8
Southeast	0	-4	-1	-2	-3	-5
Absolute Avg.	1	3	1	3	3	6
			2-Yr. Recurrence			
Northwest	+3	-3	+2	-7	-6	-8
North Central	0	-3	+1	-4	-6	-7
South Central	0	-4	+1	-4	-5	-8
Southeast	+1	-4	0	-4	-5	-6
Absolute Avg.	1	4	1	5	6	7
Over-all Average	4.2	5.0	5.0	4.2	4.2	8.6
Plus	12	9	7	6	7	13
Minus	4	10	11	11	12	7
Zero	4	1	2	3	1	0

In a further attempt to evaluate the statistical methods, a comparison was made between the several methods for the fit of the observational data to the derived curve for recurrence intervals of two years or greater, since this is the region of the curve most useful to the hydrologist. Results of this phase of the analysis are presented in Table 13 for 1-day precipitation. In this table, values for each section and for each statistical method are shown for selected recurrence intervals using annual data. The percent differences were calculated between the ranked observational data and the recurrence curve values. A positive value indicates the curve gave higher values than the ranked data. Similar calculations were made for other precipitation periods but are not shown here since results were very similar.

Except for possible elimination of the log-log method, Table 13 shows little choice between the various statistical methods. Considering all sections and intervals, Frechet shows a more even distribution of positive and negative deviations. Also the data fit is somewhat better than Gumbel, log-normal, and log-log at the extreme upper end of the curve." However, less than one percent difference occurs in the over-all average between the several methods. Consequently, after this analysis it was still not feasible to make a preferential selection of the most applicable statistical distribution.

TABLE 14

COMPARISON OF RECURRENCE INTERVAL VALUES FOR NORTHWEST SECTION

Precipitation (In.) for Given Period (Days)

Recurrence (Years)	Gumbel			Log-Normal			Frechet			Chow	Jenkinson
	1	5	10	1	5	10	1	5	10	1	1
2	2.5	3.7	4.4	2.4	3.7	4.4	2.3	3.5	4.2	2.2	2.2
5	3.5	4.9	5.8	3.4	4.8	5.7	3.2	4.7	5.6	3.1	3.3
10	4.1	5.8	6.8	4.0	5.6	6.6	4.1	5.8	6.8	4.1	4.0
20	4.8	6.6	7.8	4.5	6.3	7.4	5.1	7.0	8.2	5.1	4.9
40	5.4	7.4	8.7	5.1	7.1	8.1	6.3	8.5	9.8	6.7	6.1
100	6.2	8.4	9.9	5.9	8.0	9.0	8.3	10.8	12.4	9.2	7.8

While statistical tests had indicated no significant differences in several of the methods, examination of the calculated precipitation depths showed considerable differences in these values for long recurrence periods. Precipitation values for selected recurrence intervals are shown in Table 14 based on annual data for the Northwest Section. Reference to this table shows 40-year recurrence values ranging from 5.1 to 6.7 inches for 1-day precipitation. Similar differences can be noted for other periods and recurrence intervals in this table. Frechet, Chow, and Jenkinson give larger values for long recurrence intervals. For shorter recurrence intervals, the magnitude of the differences between the methods is not large. One cannot judge whether the the relatively large values predicted for long recurrence intervals are more or less appropriate than the smaller ones.

Although no single method emerged as the best fit to the raw data at this point, the authors favored the Frechet distribution, which gives significantly larger values than the log-normal or Gumbel methods at long recurrence intervals. This fact, of course, is not in itself a good reason for selecting the Frechet method, especially when several methods indicated approximately equal reliability in statistical tests. However, Table 13 shows some evidence in favor of the Frechet method at the upper end of the regression curves, where the differences among distributions become appreciable. The data fit with the Frechet method is illustrated in Figure 11, using 1-day annual maximum precipitation. The open circles represent individual station values and the solid circles are averages.

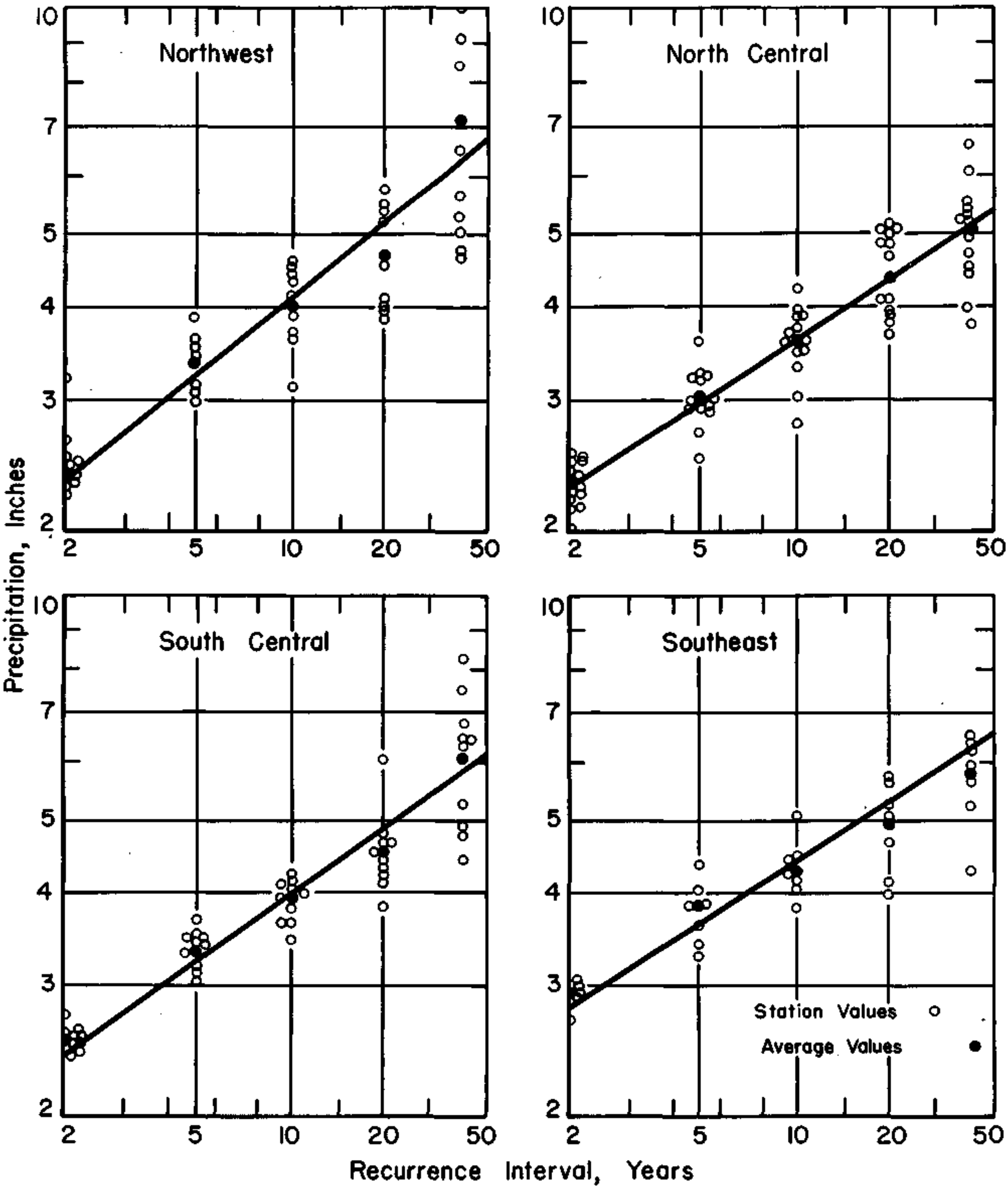

FIGURE 11 ANNUAL MAXIMA ANALYSIS OF 1-DAY PRECIPITATION FREQUENCIES

Frequency Relations Derived from Areal Analyses and Station-Year Method

Since 1948 the Water Survey has been making detailed field surveys of severe rainstorms in Illinois as part of its meteorological program.(13) It was decided to study results from analyses of these storms for indication of the magnitude, extent, and frequency of unusually heavy rainfall amounts within the state. Fortunately, the 10 years for which field survey data are available appear to provide a representative period, with six below normal years and four above normal years of precipitation in Illinois. Isohyetal maps of all severe rainstorms in Illinois for the period 1916-57, obtained from Weather Bureau climatological network data, were also available for assistance in this phase of the frequency investigation.

TABLE 15

AREAL DISTRIBUTION OF HEAVY RAINFALL IN 2-DAY STORMS BY 10-YEAR PERIODS

Area (Sq. Mi.) and Number of Storms with Rainfall

Equalling or Exceeding Given Depth (In.)

	6-inch		7-inch		8-inch		9-inch		10-inch	
Period	Area	Number	Area	Number	Area	Number	Area	Number	Area	Number
1948-57	16890	19	9425	13	5035	10	2800	9	1425	7
1938-47	10820	12	5100	4	3170	3	2360	3	1730	1
1928-37	2910	8	270	2	none	0	none	0	none	0
1918-27	12670	10	5990	6	2630	3	260	2	none	0

Basically, an areal analysis is preferable to a point analysis and certainly should provide a more realistic sampling of the magnitude and extent of rainfall at the core of severe storms. The effect of sampling density on the distribution of heavy rainfall amounts is indicated by the comparison of four 10-year periods in Table 15. In this table, the aggregate areas enclosed by isohyets ranging from 6 to 10 inches are shown for each 10-year period. Also, the number of storms in each period having amounts exceeding 6, 7, 8, 9, and 10 inches is indicated. The U. S. Weather Bureau climatological network in Illinois(3) consisted of approximately 90 gages during 1918-27 and was increased to approximately 110 gages by 1937. More rapid increases in gage density followed and by 1942 there were 160 gages in the state. This number was increased to 250 by 1948 and to 280 by 1957. As mentioned previously, detailed field surveys were conducted by the Illinois State Water Survey following the major storms of 1948-57 to provide better definition of the magnitude and distribution of rainfall in the storm centers.

Reference to Table 15 indicates a trend for both enclosed areas and number of storms to increase with increasing sampling density, except for the 1928-37 period which reflects the effects of one of the worst drought periods in the history of the Midwest. One storm in each of the two periods, 1918-27 and 1938-47, produced most of the areal coverage for isohyetal values of 7 to 10 inches; otherwise, the differences between these two periods and 1948-57 would have been much greater. It can be noted that for the 1938-47 period one storm provided more area within the 10-inch isohyet than did seven storms in 1948-57. A 2-day total of 13.77 inches and a 3-day total of 16.57 inches were recorded at Belleville in southwestern Illinois in this August 1946 storm. Field surveys such as conducted since 1948 would undoubtedly have revealed still higher values. These field surveys have shown that storms with amounts exceeding 10-12 inches are not rare occurrences in Illinois.

Consideration was given also to the possible use of the station-year method(11) in the Illinois frequency study. Use of this method requires meteorological homogeneity with respect to area, a satisfactory length of record, and independence of station observations. If all of these conditions are met, supposedly the records from several stations may be combined and treated as a single record equal to the sum of the individual records. The first two of the above requirements are satisfactorily met with the Illinois data, but many of the observations are certainly not meteorologically independent. However, if only the highest 5 to 10 percent of the precipitation values were used for each section in a station-year analysis, the requirement for independence would generally be met. Conceivably, the results of such a station-year analysis could then be used to extend the average sectional curves beyond the 40-year return period for which multiple samples were available to determine the average curves.

Although the above method may appear to be theoretically acceptable, the authors believe the station-year extension technique has a basic weakness which makes use of this method questionable in the present study. The return period values beyond the limits of the average curve would be based upon individual observations at the upper end of the average curve where the sampling is most unreliable. For example, the mean value of 1-day precipitation in the South Central Section is based upon 400 observations since there are 10 stations in this region. The 40-year value on the average frequency curve, however, is defined by only 10 observations, that is, the highest value at each station in the 40-year period. The standard deviation of the 40-year values of 1-day precipitation in the South Central Section is 1.24 inches compared to 0.26 inch at the 10-year recurrence interval and 0.09 inch at the 2-year return period. Furthermore, experience with storm field surveys during the past 10 years has emphasized the inadequacy of rainfall measurements at a few points to sample reliably the unusually severe storms which determine the long recurrence interval values. Results obtained from limited testing of the station-year method will be presented later.

In performing an areal analysis of frequency relations for Illinois, detailed isohyetal maps of 2-day severe rainstorms during 1948-57 were first used. The data for the entire state were combined in this analysis due to the relatively short period of field data available. Analysis was restricted to the area enclosed by isohyets of 6 inches or greater, where the distribution was most accurately defined as a result of the field surveys. Allowance was made for over-lapping isohyets in determining frequency distributions from the areal data.

For comparison purposes, 2-day frequency relations were next determined from data for the 20-year period, 1938-57, after adjusting the 1938-47 data using a gage density factor developed from the 1948-57 data. The gage density factor for each isohyet represented the average ratio of the area enclosed by that isohyet using both climatological network and field survey data to the area enclosed using only climatological data. Most of the severe storms during 1938-47 occurred in the latter part of this 10-year period when the climatological network was not radically different from that during 1948-57. For the purpose of further comparison which might possibly help explain the distribution of heavy rainstorms, the same gage density adjustment was applied to the 1918-37 areal data and the 40-year period, 1918-57, used to develop average 2-day frequency relations for Illinois. Obviously, the gage density was much less, and one would then expect the frequency relations developed from the 1918-57 data to provide smaller values for a given return period, due to the underestimates of high rainfall amounts and coverage.

The station-year method was investigated next and 2-day frequency relations determined for the state using the 1916-55 point rainfall data from the 39 stations available. Both the areal and station-year frequency curves were fitted by graphical analysis. The areal data closely approximated a straight-line fit on a semi-logarithmic scale and the station-year data on a log-log scale. Results from these analyses are presented in Table 16, along with values obtained from the 1916-55 point data using regression analysis to fit the data to the Frechet and Gumbel distributions.

Table 16 shows excellent agreement between the 2-day frequency relations developed from the 1948-57 and 1938-57 areal data and those obtained from the Frechet distribution. As expected, the frequencies obtained from the 1918-57 areal data are considerably below these obtained by the above methods because of inadequate number of raingages during a large part of the period. The station-year values agree closely with the 1918-57 areal values, while the Gumbel distribution provides the lowest values of all the methods, especially at the longer return periods.

TABLE 16

COMPARISON OF 2-DAY AVERAGE FREQUENCIES FOR ILLINOIS DEVELOPED BY POINT AND AREAL ANALYSIS METHODS

Analysis Method	Data Period	Rainfall (In.) Equalled or Exceeded for Given Return Period (Yrs.)			
		25	50	100	200
Areal	1948-57	6.4	7.4	8.5	9.5
Areal	1938-57	6.0	7.1	8.5	10.1
Areal	1918-57	5.5	6.4	7.4	8.6
Station-Year	1916-55	5.7	6.6	7.6	8.8
Frechet	1916-55	6.0	7.1	8.5	9.9
Gumbel	1916-55	5.6	6.3	6.8	7.5

DIFFERENCE (%) FROM FRECHET METHOD

		25	50	100	200
Areal	1948-57	+7	+4	0	-4
Areal	1938-57	0	0	0	+2
Areal	1918-57	-8	-10	-13	-13
Station-Year	1916-55	-5	-7	-11	-11
Gumbel	1916-55	-7	-11	-20	-24

TABLE 17

COMPARISON OF AVERAGE 1-DAY AND 3-DAY FREQUENCIES FOR ILLINOIS DEVELOPED BY POINT AND AREAL ANALYSIS METHODS

Analysis Method	Data Period	Rainfall (In.) Equalled or Exceeded for Given Return Period (Yrs.)			
		25	50	100	200
		1-Day Storm Period			
Areal	1948-57	5.2	6.2	7.2	8.2
Station-Year	1916-55	4.6	5.3	6.2	7.2
Frechet	1916-55	5.0	6.0	7.1	8.3
Gumbel	1916-55	4.7	5.3	5.8	6.4
		3-Day Storm Period			
Areal	1948-57	6.5	7.5	8.6	9.6
Station-Year	1916-55	6.1	7.1	8.2	9.4
Frechet	1916-55	6.6	7.9	9.5	11.0
Gumbel	1916-55	6.0	6.7	7.4	8.2

The analysis procedure described in preceding paragraphs was followed next for 1-day and 3-day storm periods, except that only 1948-57 data were used in the areal analysis since this was the period of most reliable data. No analysis was performed for 5-day and 10-day storm periods due to the lack of field survey data for these periods. Results of the 1-day and 3-day frequency analyses are presented in Table 17. Results are similar to those in Table 16 except that the Frechet values rise considerably above the 1948-57 areal values at the upper end of the curves for 3-day storms, while the station-year and areal values converge in this region of the curves. The Frechet departure at the upper end of the curves is due to the fact that very little difference existed between the 2-day and 3-day amounts in the 1948-57 severe rainstorms, while the 1916-55 point data produces an appreciable difference between these two periods.

The authors realize, of course, that the results presented in Tables 16 and 17 are based upon a limited sample of field survey data, that it is assumed the 1948-57 data represent a near normal period for severe storms, and that it is also assumed a reasonable gage density correction for 1938-47 data can be obtained from 1948-57 data. However, the results do lend support to the selection of the Frechet distribution for analysis of Illinois annual maxima data. The results of the analyses discussed in this section were the basis for the final selection of the Frechet method for this purpose.

The authors do not believe that the Frechet distribution is an unique tool for storm rainfall frequency analysis——it merely seems to provide the most realistic yardstick for analysis within the limits of the data in the present situation. The authors also believe that the Frechet curve rises too rapidly to define accurately the storm rainfall magnitude as return periods reach several hundred years, while the Gumbel and log-normal distributions rise too slowly. This opinion is based upon the analysis of the 1948-57 areal data as well as a consideration of meteorological . principles. When an additional 10 years of detailed field survey data are collected in Illinois, a considerably more reliable estimate of storm rainfall distribution in time and space will be possible.

Determination of Seasonal Relations

Seasonal frequency relations were determined by analysis procedures similar to those applied in selection of the Frechet method for the analysis of annual maxima data. Owing to the authors' belief that a statistical distribution can serve only as a yardstick for predicting future events, it was decided to choose a method for seasonal relations that would fit the data best, and not try to make the annual and seasonal data conform to the same distribution.

As a result of the analysis procedures followed it was decided to fit summer data to the Frechet distribution and winter, spring, and fall data to the log-normal distribution using the method of least squares. In all comparisons, there was little choice between the log-normal and Gumbel methods. In general, the statistical tests indicated a slightly better fit of the raw data with the log-normal method, but this difference was of little significance. The statistical tests indicated appreciably better data fit with either the log-normal or Gumbel method than with the Frechet for winter, spring, and fall. Also, the Frechet precipitation values, especially in winter, appeared to increase too rapidly with increasing recurrence intervals to satisfy climatological and meteorological considerations. In the summer when unusually heavy rainfall is most likely to occur over most of the state, the Frechet characteristics appear to fit the frequency distribution better than the other methods, especially for the longer recurrence values. The annual data which are drawn from the highest seasonal value each year also appear to fit the more rapidly rising Frechet curve best at long recurrence intervals, as pointed out earlier. A large portion of the annual values, of course, are drawn from the summer maxima. Statistical comparisons between the three methods are illustrated in Tables 18 and 19, using 1-day winter precipitation. Table 18 shows correlation coefficients for the regression equations for each station, while Table 19 shows a comparison of the raw data fit at selected recurrence intervals.

TABLE 18

CORRELATION COEFFICIENTS FOR 1-DAY WINTER REGRESSION EQUATIONS

	Correlation Coefficients for Given Section and Method			
Method	North-west	North Central	South Central	South-east
Gumbel	0.97	0.95	0.93	0.97
Log-Normal	0.98	0.97	0.94	0.98
Frechet	0.96	0.95	0.94	0.96

Determination of Confidence Limits

Methods in current use for computing confidence limits for probability curves have been designed for a single period of record at a single station. For the present analysis, data for the same period of record at several stations were combined to obtain an average probability curve for points in an area.

TABLE 19

PERCENT DIFFERENCE BETWEEN RANKED DATA AND REGRESSION CURVE VALUES FOR 1-DAY WINTER FREQUENCIES

	Difference (%) for Given Section and Recurrence				
Method	Northwest	North Central	South Central	Southeast	Absolute Average
		40-Yr. Recurrence			
Gumbel	-10	-5	-5	-3	6
Log-Normal	-9	-3	-6	+1	5
Frechet	+13	+22	+18	+31	21
		20-Yr. Recurrence			
Gumbel	-4	-6	-6	+1	4
Log-Normal	-5	-6	-6	+1	5
Frechet	+9	+8	+5	+18	10
		10-Yr. Recurrence			
Gumbel	-2	-3	-3	-4	3
Log-Normal	-2	-3	-5	-4	4
Frechet	+1	-1	-2	-1	1
		5-Yr. Recurrence			
Gumbel	+3	+6	+2	+3	4
Log-Normal	+2	+4	0	+5	3
Frechet	-3	-1	-4	-4	3
		2-Yr. Recurrence			
Gumbel	+1	0	+2	+1	1
Log-Normal	0	-1	-1	-2	1
Frechet	-5	-8	-6	-9	7

Combining station data introduces an additional variance factor at each rank which is not accounted for in theoretical confidence limit systems.

Confidence limits which were based on the standard error of estimate from a least-squares fitting process were essentially parallel to the regression curve. The confidence region was too wide in the region of the mean value and too narrow near the data extremes to define adequately the data range. Obviously, the accuracy of predicted extremes is much better in the region of the mean than near ends of the regression curve.

Examination of the data indicated that the standard deviation increased rapidly in magnitude as the rainfall recurrence interval increased. Therefore, it was decided that the range of variability about the regression curve could be defined best by computing standard deviations at numerous recurrence intervals along the regression curve. Logarithms of rainfall were used to achieve approximate normality in determining the standard deviations.

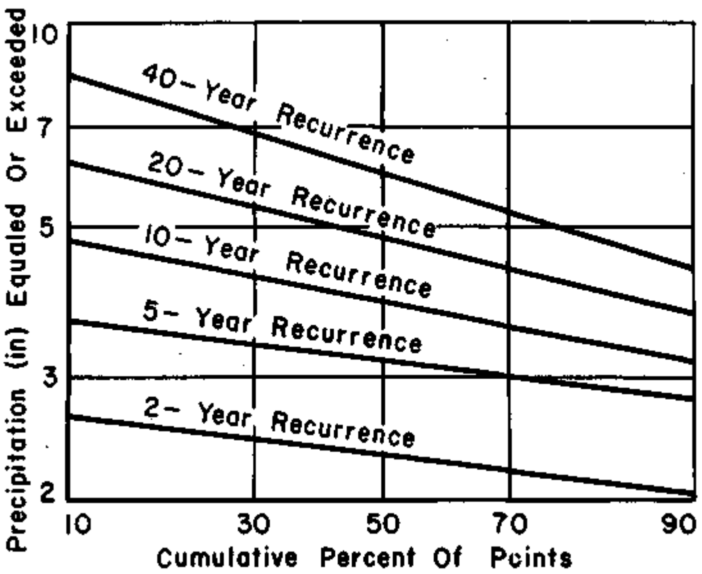

FIGURE 12 PRECIPITATION DISTRIBUTION AT SELECTED RECURRENCE INTERVALS FOR 1-DAY STORMS IN NORTHWEST SECTION

Standard deviation calculations were used in conjunction with regression curve values at various recurrence intervals to construct curves such as those illustrated in Figure 12. In this figure the cumulative percent of points has been related to precipitation at various recurrence intervals for 1-day precipitation in the Northwest Section. Curves such as those shown in Figure 12, in turn, were used to obtain probability curves illustrated in Figure 13. Probability levels of 10 to 90 percent were determined for each section and each precipitation period by the method described.

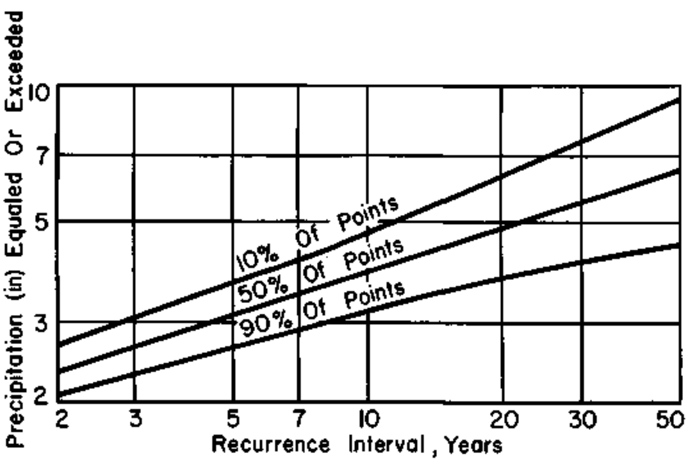

FIGURE 13 PROBABILITY CURVES FOR 1-DAY PRECIPITATION IN NORTHWEST SECTION

Relation Between Annual Maxima and Partial Duration Series

The partial duration series includes all of the high values occurring within a period, whereas the annual maxima includes only the highest value of each particular year within a period. The partial duration series permit use of more than one value in a particular year. Thus, the 50 highest values occurring in a 50-year period will always be included in the partial duration series, but not necessarily among the annual maxima, since the second or third highest values in some years may be greater than the highest value in certain other years. However, processing of partial duration data is very laborious compared to processing of annual maxima data.

Since partial duration values are frequently more useful than those from the annual series, empirical factors are frequently developed to transform annual series values to partial duration values. The U. S. Weather Bureau, for example, has used a selected sample of 50 precipitation stations throughout the United States to develop such transformation factors.(14) As a result of its study, the Weather Bureau found that the ratio of partial duration to annual series values averaged 1.13 for a 2-year return period, 1.04 for a 5-year return period, and 1.01 for a 10-year return period. Beyond a 10-year period, it found the differences between the two series to be insignificant. The Weather Bureau transformation factors apply to periods of 5 minutes to 24 hours.

As part of the Illinois study, data from 10 stations distributed throughout the state were analyzed to obtain average transformation factors for storm periods of one to 10 days. The results of this analysis are summarized in Table 20.

TABLE 20

RATIO OF PARTIAL DURATION TO ANNUAL MAXIMA FREQUENCIES

Precipitation Period (Days)	Ratio for Given Recurrence Interval (Yrs.) 2	5	10
1	1.13	1.05	1.01
2	1.09	1.02	1.01
5	1.09	1.01	1.01
10	1.08	1.01	1.00

The 1-day transformation factors for Illinois in Table 20 are almost identical with those found, in general, for the United States by the Weather Bureau. The ratios decrease slightly with increasing storm period, according to the Illinois study. Comparison of these empirical transformation factors with similar theoretically derived factors(11) shows smallerratios for the empirical factors. Chow indicates approximate ratios of 1.21, 1.10, and 1.05 for recurrence intervals of 2, 5, and 10 years, respectively, from theoretical considerations. The transformation factors of Table 20 should be used for converting the Illinois maxima value's of Tables 23 to 32 to partial duration frequencies. Differences are insignificant for recurrence intervals greater than 10 years.

Comparison Between Calendar-Day and Maximum Period Amounts

To incorporate a large number of stations into the Illinois frequency study, it was necessary to use data from Weather Bureau cooperative stations. Since the majority of these stations have non-recording gages, the frequency analysis has been made on a calendar-day basis. For example, the 1-day frequency relations are based upon 24-hour precipitation totals ending at the time of daily observation at each station. The daily total, of course, does not necessarily include the 24-hour maximum total in a given storm.

Data from six recording gages distributed throughout the state were analyzed for the 10-year period, 1948-57, to determine transformation factors for converting calendar-day to maximum period precipitation. Stations used for this purpose were Rockford, Peoria, Paris, Effingham, Belleville, and Cairo. The transformation factors represent the average ratio of maximum 24-hour, 48-hour, 72-hour, 120-hour, and 240-hour precipitation to 1-day, 2-day, 3-day, 5-day, and 10-day calendar-day amounts, respectively. The ratios are based upon storm periods with precipitation equalling or exceeding 1.5, 2.0, 2.5, 3.0, and 4.0 inches, respectively, for 1, 2, 3, 5, and 10 days. These precipitation values included all those equalling or exceeding the 2-year return period value for the various storm periods, but eliminated the effects of relatively light storms of little interest on calculated transformation factors.

No distinct trend that the transformation factors varied over the state was noted, and as the variability of the factors among stations was sufficiently small, an analysis of a larger sample was not deemed necessary at this time. Furthermore, the factor for converting from 24-hour calendar-day to 24-hour maximum precipitation is the same as that obtained by the U. S. Weather Bureau in other regions of the United States.(14) Results of this analysis are summarized in Table 21. The transformation factors in this table may be used for converting the calendar-day values of Tables 23 to 32 to maximum period precipitation. Except for 1-day storms, however, the differences between calendar-day and maximum period values are hardly significant.

TABLE 21

RATIO OF MAXIMUM PERIOD TO CALENDAR-DAY PRECIPITATION

Storm Period (Days)	Ratio
1	1.13
2	1.02
3	1.01
5	1.01
10	1.01

Duration of Precipitation During Storm Periods of 1 to 10 Days

In problems of utilizing precipitation frequency data for hydrologic design, the question frequently arises regarding the actual duration of rainfall in given storm periods. For example, the 24-hour rainfalls from which a frequency relation is derived may include precipitation periods encompassing a few minutes up to 24 hours. To help explain this problem, the 1948-57 data for six recording stations, discussed in the previous section, were analyzed to determine the actual duration of precipitation during storm periods of various length. Hourly precipitation data from Weather Bureau climatological publications were used in the study.

In the first phase of this analysis, precipitation periods of 1 to 3 days were examined to determine the average number of hours between the beginning and ending of precipitation. Results show that 24-hour rainfall totals on the average occur within a period of 14 hours. That is, the average time from the beginning to the end of the over-all storm is 14 hours, although the actual duration of rainfall may be considerably less than 14 hours, since the storm may be made up of several individual showers within this period. The average number of hours with rain in a storm period will be discussed in the following paragraphs. The average extent of 2-day storms was found to be 30 hours, while 3-day storms averaged 45 hours from beginning to end.

In the second phase of the analysis, the average number of hours having precipitation during storm periods of 1 to 10 days was investigated. Results are summarized in Table 22 which shows that while the average 24-hour storm encompasses a period of 14 hours, precipitation is recorded in only 10 of the 14 hours. Similarly, only 17 of 30 hours have precipitation during 2-day storms and 22 out of 45 hours in 3-day storms. The results of Table 22 may be used with the frequency relations presented in Tables 23 to 32 when a more detailed time distribution is required. Although no distinct trend was noted over the state in the average length of storm period, some evidence was found of a tendency for the number of hours with precipitation in storms to increase southward.

TABLE 22

PRECIPITATION DURATION FOR STORM PERIODS OF 1 TO 10 DAYS

	Storm Period (Days)				
	1	2	3	5	10
Avg. length of precipitation period (hours)	14	30	45	--	--
Avg. number of hours with precipitation	10	17	22	26	32

Frequencies for 1 to 10 Days from Annual and Seasonal Maxima

Final results of the analysis of Illinois frequency relations for storm periods of 1 to 10 days on an annual and seasonal basis are presented in Tables 23 to 32. These results are based upon analysis of the annual maxima series; that is, the annual frequency relations were determined from an analysis of the maximum value recorded in each of the *40* years, 1916-55, while the seasonal frequency relations were based on the maximum value for each particular season in the 40-year period. The frequency relations are also based upon analysis of calendar-day data. Transformation factors for converting from calendar-day to maximum period precipitation and from annual series to partial duration series values are presented in Tables 21 and 20, respectively. Intervals between tabulated values in Tables 23 to 32 are sufficiently small to allow linear interpolation for obtaining recurrence intervals that are not listed. Values given for recurrence intervals of 75 and 100 years in Table 23 should be used with caution, since they were obtained by extrapolating well beyond the limits of the data sample.

TABLE 23

AVERAGE FREQUENCY OF ANNUAL MAXIMUM PRECIPITATION, 1-DAY TO 10-DAY AMOUNTS

Northwest Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	2.3	2.8	3.1	3.5	4.2
3	2.7	3.2	3.6	4.1	4.8
5	3.2	3.8	4.2	4.8	5.7
10	3.9	4.7	5.2	5.9	7.0
15	4.4	5.3	5.9	6.6	7.8
20	4.8	5.7	6.4	7.2	8.6
25	5.1	6.1	6.8	7.7	9.2
30	5.5	6.5	7.2	8.1	9.7
40	5.9	6.9	7.7	8.7	10.4
50	6.4	7.4	8.3	9.3	11.3
75	7.0	8.4	9.3	10.5	12.6
100	7.5	9.0	9.9	11.2	13.2

North Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	2.3	2.7	3.0	3.4	4.1
3	2.6	3.1	3.5	3.9	4.7
5	3.0	3.6	4.0	4.5	5.4
10	3.6	4.3	4.8	5.5	6.6
15	4.0	4.8	5.3	6.0	7.3
20	4.3	5.2	5.8	6.6	7.9
25	4.6	5.5	6.1	6.9	8.4
30	4.8	5.7	6.4	7.2	8.7
40	5.1	6.1	6.8	7.8	9.4
50	5.4	6.5	7.2	8.1	9.8
75	6.0	7.2	8.0	9.0	10.9
100	6.4	7.7	8.6	9.8	11.7

TABLE 23 (Cont'd)

South Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days)				
	1	2	3	5	10
2	2.4	2.9	3.3	3.8	4.5
3	2.7	3.2	3.6	4.1	5.0
5	3.3	3.9	4.4	5.1	6.2
10	4.0	4.8	5.4	6.2	7.6
15	4.5	5.4	6.0	6.8	8.3
20	4.9	5.9	6.6	7.5	9.2
25	5.2	6.2	6.9	7.9	9.7
30	5.4	6.6	7.3	8.4	10.2
40	5.8	7.0	7.8	9.1	10.8
50	6.2	7.4	8.3	9.4	11.6
75	6.9	8.3	9.2	10.5	12.9
100	7.4	8.9	9.9	11.4	13.8

Southeast Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days)				
	1	2	3	5	10
2	2.8	3.3	3.7	4.2	5.1
3	3.1	3.7	4.1	4.7	5.6
5	3.7	4.4	4.9	5.6	6.7
10	4.4	5.3	5.9	6.7	8.0
15	4.9	5.9	6.5	7.4	8.9
20	5.3	6.3	7.0	8.0	9.6
25	5.6	6.7	7.5	8.4	10.2
30	5.8	7.0	7.7	8.7	10.6
40	6.2	7.5	8.3	9.3	11.3
50	6.5	7.7	8.5	9.6	11.6
75	7.3	8.7	9.7	11.0	13.2
100	7.8	9.3	10.3	11.8	14.2

The data in Tables 24 to 28 provide a measure of the areal variability of point precipitation within the four sections at selected recurrence intervals for storm periods of 1 to 10 days. Assuming sectional homogeneity, Tables 24 to 28 also provide probability estimates at the 10 to 90 percent levels for any given point. The use of these tables will be illustrated by an example from Table 24. Referring to the 20-year recurrence interval column for the Northwest Section, it is seen that 10 percent of the points (and consequently 10 percent of the sectional area) will have a 1-day precipitation total equalling or exceeding 6.3 inches, while 50 and 90 percent of the points will have amounts equalling or exceeding 4.8 and 3-8 inches, respectively, in an average 20-year period. Since the section is assumed to be meteorologically homogeneous, the percentages in column 1 of Tables 24 to 28 should also represent the probability of any point having precipitation equalling or exceeding the corresponding depth given in the recurrence interval columns. Thus, again referring to the 20-year recurrence column for the Northwest Section in Table 24, it is seen that at any given point in this section, there is a 10 percent probability (one chance in 10) that the 1-day maximum precipitation will equal or exceed 6.3 inches in a 20-year period. Similarly, there is a 50 percent probability that the precipitation at a selected point will equal or exceed 4.8 inches and a 90 percent probability that it will be 3.8 inches or greater in a 20-year period.

TABLE 24

AREAL FREQUENCY DISTRIBUTION OF ANNUAL MAXIMUM PRECIPITATION, 1-DAY AMOUNTS

Northwest Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.) 50	40	30	20	10	5	2
10	9.4	8.5	7.5	6.3	4.8	3.7	2.7
20	8.5	7.7	7.0	5.9	4.5	3.5	2.6
30	7.7	7.1	6.4	5.5	4.3	3.4	2.5
40	7.1	6.5	6.0	5.1	4.1	3.3	2.4
50	6.4	5.9	5.5	4.8	3.9	3.2	2.3
60	6.0	5.6	5.2	4.6	3.7	3.1	2.3
70	5.6	5.3	4.9	4.4	3.6	3.0	2.2
80	5.1	4.8	4.5	4.0	3.4	2.9	2.2
90	4.6	4.4	4.1	3.8	3.2	2.7	2.1

North Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.) 50	40	30	20	10	5	2
10	7.0	6.6	6.0	5.2	4.2	3.4	2.6
20	6.4	6.1	5.5	4.9	4.0	3.3	2.5
30	5.9	5.6	5.1	4.6	3.8	3.2	2.5
40	5.7	5.3	4.9	4.5	3.7	3.1	2.4
50	5.4	5.1	4.8	4.3	3.6	3.0	2.3
60	5.2	4.9	4.6	4.2	3.5	2.9	2.3
70	5.0	4.7	4.5	4.0	3.4	2.9	2.2
80	4.8	4.5	4.3	3.9	3.3	2.8	2.2
90	4.6	4.4	4.1	3.7	3.2	2.7	2.1

TABLE 24 (Cont'd)

South Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	7.4	7.0	6.3	5.6	4.5	3.6	2.7
20	7.1	6.7	6.0	5.4	4.3	3.5	2.6
30	6.8	6.4	5.8	5.2	4.2	3.4	2.5
40	6.4	6.0	5.6	5.1	4.1	3.3	2.5
50	6.2	5.8	5.4	4.9	4.0	3.3	2.4
60	5.9	5.6	5.2	4.7	3.8	3.2	2.4
70	5.7	5.4	5.0	4.5	3.7	3.1	2.4
80	5.4	5.1	4.8	4.3	3.6	3.1	2.3
90	5.1	4.8	4.6	4.1	3.5	3.0	2.3

Southeast Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	7.7	7.2	6.7	6.0	4.9	4.1	3.1
20	7.4	6.9	6.4	5.7	4.7	4.0	3.0
30	7.1	6.7	6.2	5.5	4.6	3.9	2.9
40	6.9	6.4	6.0	5.4	4.5	3.8	2.9
50	6.6	6.2	5.8	5.3	4.4	3.7	2.8
60	6.3	5.9	5.5	5.0	4.2	3.6	2.8
70	6.0	5.7	5.3	4.8	4.1	3.5	2.7
80	5.6	5.3	5.0	4.5	3.9	3.4	2.7
90	5.2	4.9	4.7	4.3	3.8	3.3	2.6

TABLE 25

AREAL FREQUENCY DISTRIBUTION OF 2-DAY PRECIPITATION AMOUNTS

Northwest Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.) 50	40	30	20	10	5	2
10	10.0	9.3	8.3	7.1	5.5	4.3	3.0
20	9.4	8.7	7.8	6.7	5.2	4.1	2.9
30	8.7	8.1	7.3	6.4	5.0	4.0	2.9
40	8.0	7.5	6.9	6.0	4.8	3.9	2.9
50	7.4	6.9	6.5	5.7	4.7	3.8	2.8
60	7.0	6.5	6.2	5.5	4.5	3.7	2.8
70	6.6	6.2	5.9	5.3	4.3	3.6	2.8
80	6.2	5.7	5.4	4.9	4.1	3.4	2.7
90	5.7	5.3	5.0	4.6	3.9	3.3	2.7

North Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.) 50	40	30	20	10	5	2
10	8.0	7.4	6.8	6.0	5.0	4.1	3.0
20	7.5	7.0	6.4	5.7	4.7	3.9	3.0
30	7.0	6.6	6.1	5.5	4.5	3.8	2.9
40	6.7	6.3	5.9	5.3	4.4	3.7	2.8
50	6.5	6.1	5.7	5.2	4.3	3.6	2.7
60	6.3	5.9	5.5	5.0	4.2	3.5	2.7
70	6.1	5.7	5.4	4.9	4.1	3.4	2.6
80	5.9	5.5	5.2	4.7	4.0	3.4	2.6
90	5.7	5.3	5.1	4.6	3.9	3.3	2.5

TABLE 25 (Cont'd)

South Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	11.4	10.4	9.4	8.0	6.1	4.6	3.3
20	9.9	9.2	8.6	7.3	5.6	4.4	3.2
30	9.0	8.3	7.7	6.6	5.2	4.3	3.1
40	8.2	7.6	7.0	6.2	5.0	4.1	3.0
50	7.4	7.0	6.6	5.9	4.8	3.9	2.9
60	7.0	6.6	6.2	5.6	4.6	3.8	2.9
70	6.7	6.3	5.9	5.4	4.5	3.8	2.8
80	6.3	5.9	5.6	5.1	4.3	3.7	2.8
90	6.0	5.6	5.3	4.8	4.2	3.6	2.8

Southeast Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	9.4	8.9	8.3	7.4	6.3	5.2	4.1
20	9.0	8.5	7.9	7.1	6.0	5.0	3.9
30	8.7	8.2	7.6	6.9	5.7	4.8	3.7
40	8.3	7.8	7.3	6.6	5.5	4.6	3.5
50	7.9	7.5	7.0	6.3	5.3	4.4	3.3
60	7.6	7.2	6.7	6.1	5.1	4.3	3.3
70	7.4	7.0	6.5	5.9	5.0	4.2	3.3
80	7.2	6.7	6.2	5.7	4.8	4.1	3.2
90	6.8	6.3	6.0	5.5	4.6	4.0	3.2

TABLE 26

AREAL FREQUENCY OF ANNUAL MAXIMUM PRECIPITATION, 3-DAY AMOUNTS

Northwest Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	11.4	10.5	9.4	8.0	6.2	4.8	3.3
20	10.6	9.8	8.8	7.6	5.9	4.5	3.3
30	9.8	9.1	8.2	7.2	5.6	4.4	3.2
40	9.0	8.4	7.6	6.8	5.4	4.3	3.2
50	8.3	7.7	7.2	6.4	5.2	4.2	3.1
60	7.7	7.2	6.7	6.0	4.9	4.0	3.1
70	7.2	6.8	6.3	5.7	4.7	3.9	3.0
80	6.7	6.3	5.9	5.3	4.5	3.8	3.0
90	6.2	5.9	5.5	5.0	4.3	3.7	2.9

North Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	8.5	8.0	7.4	6.4	5.5	4.5	3.4
20	8.1	7.7	7.1	6.2	5.2	4.3	3.3
30	7.8	7.4	6.8	6.1	5.0	4.2	3.2
40	7.5	7.1	6.6	5.9	4.9	4.1	3.1
50	7.2	6.8	6.4	5.8	4.8	4.0	3.0
60	7.0	6.6	6.1	5.6	4.6	3.9	3.0
70	6.8	6.4	5.9	5.4	4.5	3.8	2.9
80	6.5	6.2	5.7	5.2	4.3	3.7	2.9
90	6.2	5.7	5.4	4.9	4.2	3.6	2.8

TABLE 26 (Cont'd)

South Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	12.4	11.4	10.1	8.7	6.6	5.1	3.6
20	11.0	10.2	9.2	7.9	6.2	4.9	3.5
30	9.7	9.0	8.3	7.2	5.8	4.7	3.5
40	8.9	8.4	7.8	6.9	5.6	4.5	3.4
50	8.3	7.8	7.3	6.6	5.4	4.4	3.3
60	7.9	7.4	6.9	6.3	5.2	4.3	3.3
70	7.5	7.0	6.6	6.0	5.0	4.2	3.2
80	7.0	6.4	6.1	5.5	4.7	4.0	3.2
90	6.4	6.0	5.7	5.2	4.5	3.9	3.1

Southeast Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	10.0	9.4	8.8	7.9	6.6	5.5	4.3
20	9.6	9.1	8.5	7.6	6.4	5.3	4.1
30	9.3	8.8	8.2	7.4	6.2	5.2	4.0
40	8.9	8.5	7.9	7.2	6.0	5.0	3.8
50	8.5	8.3	7.7	7.0	5.9	4.9	3.7
60	8.3	8.1	7.5	6.8	5.7	4.8	3.7
70	8.1	7.9	7.3	6.6	5.6	4.7	3.6
80	7.8	7.6	7.1	6.4	5.4	4.6	3.6
90	7.6	7.3	6.9	6.3	5.3	4.5	3.5

TABLE 27

AREAL FREQUENCY OF ANNUAL MAXIMUM PRECIPITATION, 5-DAY AMOUNTS

Northwest Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	12.0	11.0	9.9	8.6	6.8	5.4	3.7
20	11.1	10.3	9.3	8.1	6.5	5.2	3.7
30	10.3	9.6	8.8	7.7	6.3	5.1	3.6
40	9.8	9.1	8.4	7.4	6.1	5.0	3.6
50	9.3	8.7	8.1	7.2	5.9	4.8	3.5
60	8.7	8.1	7.6	6.7	5.6	4.6	3.5
70	8.1	7.6	7.1	6.3	5.3	4.4	3.4
80	7.4	7.0	6.6	5.9	5.0	4.2	3.4
90	6.8	6.4	6.1	5.5	4.8	4.1	3.3

North Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	10.1	9.5	8.7	7.6	6.2	5.1	3.8
20	9.6	9.0	8.3	7.3	5.9	4.8	3.7
30	9.1	8.6	7.9	7.0	5.8	4.7	3.6
40	8.6	8.2	7.5	6.8	5.6	4.6	3.5
50	8.1	7.8	7.2	6.6	5.5	4.5	3.4
60	7.8	7.4	6.9	6.3	5.3	4.4	3.4
70	7.5	7.1	6.6	6.0	5.1	4.3	3.3
80	7.1	6.7	6.3	5.7	4.9	4.2	3.3
90	6.6	6.3	5.9	5.4	4.7	4.1	3.2

TABLE 27 (Cont'd)

South Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	13.0	12.0	10.8	9.4	7.3	5.6	4.1
20	12.0	11.1	10.1	8.8	7.0	5.4	4.0
30	11.0	10.2	9.4	8.3	6.7	5.3	3.9
40	10.2	9.6	8.9	7.9	6.4	5.2	3.9
50	9.4	9.1	8.4	7.5	6.2	5.1	3.8
60	8.8	8.5	7.9	7.0	5.9	4.9	3.8
70	8.2	7.9	7.4	6.6	5.6	4.7	3.7
80	7.5	7.1	6.7	6.1	5.3	4.5	3.7
90	6.7	6.4	6.0	5.6	5.0	4.4	3.6

Southeast Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	11.4	10.8	10.0	8.9	7.4	6.0	4.7
20	10.9	10.3	9.6	8.6	7.2	5.9	4.6
30	10.4	9.9	9.3	8.4	7.0	5.8	4.5
40	10.0	9.6	9.0	8.2	6.8	5.7	4.4
50	9.6	9.3	8.7	8.0	6.7	5.6	4.3
60	9.2	8.9	8.4	7.7	6.5	5.4	4.3
70	8.8	8.5	8.0	7.4	6.3	5.3	4.2
80	8.5	8.1	7.7	7.0	6.0	5.1	4.2
90	8.1	7.7	7.3	6.7	5.8	5.0	4.1

TABLE 28

AREAL FREQUENCY DISTRIBUTION OF
ANNUAL MAXIMUM PRECIPITATION, 10-DAY AMOUNTS

Northwest Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	13.7	12.8	11.5	10.0	7.9	6.2	4.6
20	13.0	12.1	10.9	9.6	7.6	6.0	4.4
30	12.3	11.5	10.5	9.2	7.4	5.9	4.3
40	11.8	10.9	10.1	8.9	7.2	5.8	4.3
50	11.3	10.4	9.7	8.6	7.0	5.7	4.2
60	10.6	9.8	9.1	8.2	6.7	5.5	4.2
70	9.8	9.2	8.5	7.8	6.4	5.4	4.1
80	9.0	8.5	7.9	7.3	6.1	5.2	4.1
90	8.2	7.8	7.4	6.8	5.8	5.0	4.0

North Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	12.3	11.4	10.4	9.0	7.4	6.0	4.5
20	11.6	10.8	9.9	8.7	7.2	5.8	4.4
30	10.9	10.2	9.5	8.4	7.0	5.6	4.3
40	10.3	9.8	9.1	8.1	6.8	5.5	4.2
50	9.8	9.4	8.7	7.9	6.6	5.4	4.1
60	9.4	9.0	8.3	7.5	6.4	5.3	4.1
70	9.0	8.5	7.9	7.1	6.2	5.2	4.0
80	8.3	7.9	7.4	6.7	5.9	5.0	4.0
90	7.5	7.2	6.8	6.3	5.6	4.8	3.9

TABLE 28 (Cont'd)

South Central Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	14.8	13.7	12.4	10.7	8.4	6.7	4.9
20	14.0	13.0	11.8	10.3	8.2	6.5	4.8
30	13.3	12.4	11.3	9.9	8.0	6.4	4.7
40	12.4	11.6	10.7	9.5	7.8	6.3	4.6
50	11.6	10.8	10.2	9.2	7.6	6.2	4.5
60	11.2	10.4	9.8	8.8	7.3	6.0	4.5
70	10.8	10.1	9.4	8.4	7.1	5.8	4.4
80	10.3	9.6	8.9	8.0	6.9	5.6	4.4
90	9.7	9.1	8.5	7.6	6.7	5.5	4.3

Southeast Section

Percent of Area	Precipitation (In.) Equalled or Exceeded for Given Recurrence Interval (Yrs.)						
	50	40	30	20	10	5	2
10	14.0	12.8	11.9	10.7	8.9	7.5	5.8
20	13.3	12.4	11.5	10.4	8.6	7.3	5.6
30	12.6	12.0	11.2	10.1	8.4	7.1	5.5
40	12.1	11.6	10.9	9.8	8.2	6.9	5.3
50	11.6	11.3	10.6	9.6	8.0	6.7	5.1
60	11.3	11.0	10.3	9.3	7.8	6.5	5.0
70	11.1	10.7	10.0	9.0	7.6	6.4	4.9
80	10.8	10.3	9.7	8.7	7.4	6.3	4.8
90	10.5	9.9	9.3	8.4	7.2	6.1	4.7

TABLE 29

AVERAGE FREQUENCY OF WINTER MAXIMUM PRECIPITATION,
1-DAY TO 10-DAY AMOUNTS

Northwest Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	0.9	1.1	1.2	1.4	1.6
3	1.1	1.3	1.4	1.6	1.9
5	1.3	1.6	1.7	1.9	2.3
10	1.6	1.9	2.1	2.3	2.7
15	1.7	2.1	2.3	2.5	2.9
20	1.8	2.2	2.4	2.7	3.1
25	1.9	2.3	2.5	2.8	3.3
30	2.0	2.4	2.6	2.9	3.4
40	2.1	2.5	2.7	3.1	3.6
50	2.2	2.6	2.8	3.2	3.7

North Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.0	1.2	1.3	1.5	1.7
3	1.2	1.4	1.6	1.8	2.1
5	1.5	1.7	1.9	2.2	2.5
10	1.8	2.1	2.3	2.7	3.1
15	2.0	2.3	2.6	2.9	3.4
20	2.1	2.5	2.8	3.1	3.7
25	2.2	2.6	2.9	3.2	3.8
30	2.3	2.7	3.0	3.4	4.0
40	2.5	2.9	3.2	3.6	4.3
50	2.6	3.0	3.3	3.7	4.5

TABLE 29 (Cont'd)

South Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days)				
	1	2	3	5	10
2	1.3	1.5	1.7	1.9	2.2
3	1.5	1.8	2.0	2.3	2.7
5	1.9	2.2	2.5	2.8	3.3
10	2.3	2.7	3.0	3.4	4.1
15	2.5	3.0	3.3	3.7	4.5
20	2.7	3.2	3.6	4.0	4.8
25	2.8	3.4	3.8	4.3	5.0
30	2.9	3.5	3.9	4.5	5.2
40	3.1	3.7	4.1	4.7	5.6
50	3.2	3.9	4.3	4.9	5.8

Southeast Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days)				
	1	2	3	5	10
2	1.7	2.1	2.3	2.7	3.2
3	2.1	2.6	2.9	3.4	4.1
5	2.6	3.2	3.5	4.1	4.9
10	3.3	4.0	4.4	5.1	6.2
15	3.6	4.4	4.9	5.6	6.8
20	3.9	4.7	5.3	6.1	7.4
25	4.1	5.0	5.6	6.5	7.9
30	4.3	5.2	5.8	6.8	8.3
40	4.6	5.6	6.2	7.2	8.7
50	4.8	5.9	6.5	7.5	9.2

TABLE 30

AVERAGE FREQUENCY OF SPRING MAXIMUM PRECIPITATION, 1-DAY TO 10-DAY AMOUNTS

Northwest Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.4	1.7	2.0	2.3	2.8
3	1.7	2.1	2.4	2.8	3.4
5	2.0	2.5	2.8	3.2	4.0
10	2.4	3.0	3.3	3.9	4.8
15	2.6	3.2	3.6	4.2	5.2
20	2.8	3.4	3.9	4.5	5.5
25	2.9	3.6	4.0	4.7	5.8
30	3.0	3.7	4.2	4.9	6.0
40	3.2	3.9	4.4	5.1	6.3
50	3.3	4.1	4.6	5.3	6.5

North Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.5	1.8	2.1	2.4	2.9
3	1.8	2.2	2.5	2.9	3.5
5	2.1	2.6	2.9	3.4	4.1
10	2.5	3.1	3.5	4.0	5.0
15	2.7	3.4	3.8	4.4	5.4
20	2.9	3.6	4.0	4.7	5.7
25	3.0	3.8	4.2	4.9	6.0
30	3.1	3.9	4.4	5.1	6.2
40	3.3	4.1	4.6	5.3	6.6
50	3.4	4.2	4.8	5.5	6.8

TABLE 30 (Cont'd)

South Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.6	2.0	2.2	2.6	3.2
3	1.9	2.4	2.7	3.2	3.9
5	2.3	2.9	3.2	3.8	4.7
10	2.8	3.5	4.0	4.6	5.7
15	3.1	3.8	4.3	5.0	6.2
20	3.3	4.1	4.6	5.4	6.7
25	3.5	4.3	4.9	5.7	7.0
30	3.6	4.5	5.1	5.9	7.3
40	3.9	4.8	5.4	6.3	7.7
50	4.0	5.0	5.6	6.5	8.0

Southeast Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	2.0	2.4	2.7	3.1	3.7
3	2.4	2.9	3.3	3.8	4.5
5	2.8	3.5	3.9	4.5	5.4
10	3.5	4.2	4.7	5.5	6.6
15	3.8	4.6	5.1	6.0	7.2
20	4.1	4.9	5.5	6.4	7.8
25	4.3	5.2	5.8	6.7	8.2
30	4.4	5.4	6.1	7.0	8.5
40	4.7	5.7	6.4	7.4	9.0
50	4.8	6.0	6.7	7.7	9.4

TABLE 31

AVERAGE FREQUENCY OF SUMMER MAXIMUM PRECIPITATION, 1-DAY TO 10-DAY AMOUNTS

Northwest Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.8	2.2	2.4	2.8	3.3
3	2.2	2.7	3.0	3.4	4.0
5	2.7	3.2	3.6	4.1	4.8
10	3.5	4.2	4.6	5.3	6.3
15	4.0	4.9	5.3	6.1	7.1
20	4.5	5.4	6.0	6.8	7.9
25	4.8	5.8	6.4	7.3	8.5
30	5.1	6.2	6.8	7.7	9.0
40	5.7	6.8	7.5	8.6	10.2
50	6.0	7.2	7.9	9.0	10.6

North Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.7	2.0	2.2	2.6	3.1
3	2.1	2.5	2.7	3.2	3.7
5	2.5	3.0	3.3	3.8	4.5
10	3.2	3.9	4.3	4.9	5.8
15	3.7	4.4	4.9	5.6	6.6
20	4.1	4.9	5.4	6.2	7.4
25	4.4	5.3	5.9	6.7	7.9
30	4.6	5.7	6.3	7.2	8.4
40	5.0	6.0	6.7	7.7	9.2
50	5.3	6.4	7.1	8.1	9.7

TABLE 31 (Cont'd)

South Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.7	2.1	2.3	2.6	3.1
3	2.1	2.5	2.8	3.2	3.8
5	2.6	3.1	3.4	3.9	4.7
10	3.4	4.1	4.5	5.2	6.2
15	3.9	4.7	5.2	6.0	7.2
20	4.4	5.3	5.9	6.7	8.1
25	4.8	5.7	6.4	7.2	8.6
30	5.1	6.1	6.8	7.6	9.1
40	5.7	6.8	7.6	8.6	10.3
50	6.1	7.2	8.2	9.2	11.0

Southeast Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.7	2.1	2.3	2.6	3.1
3	2.1	2.5	2.8	3.2	3.8
5	2.6	3.1	3.5	3.9	4.7
10	3.5	4.2	4.6	5.3	6.3
15	4.1	4.9	5.4	6.1	7.3
20	4.6	5.5	6.1	6.9	8.3
25	5.0	6.0	6.6	7.4	8.9
30	5.3	6.4	7.0	7.9	9.5
40	5.9	7.0	7.8	8.8	10.7
50	6.2	7.4	8.3	9.3	11.3

TABLE 32

AVERAGE FREQUENCY OF FALL MAXIMUM PRECIPITATION, 1-DAY TO 10-DAY AMOUNTS

Northwest Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.7	2.0	2.2	2.5	2.9
3	2.0	2.4	2.7	3.0	3.6
5	2.5	2.9	3.3	3.7	4.4
10	3.1	3.7	4.0	4.6	5.4
15	3.4	4.0	4.4	5.0	6.0
20	3.7	4.3	4.8	5.4	6.4
25	3.9	4.5	5.0	5.7	6.7
30	4.1	4.7	5.2	5.9	7.0
40	4.3	5.1	5.6	6.3	7.5
50	4.5	5.3	5.8	6.6	7.8

North Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.6	1.9	2.1	2.4	2.9
3	1.9	2.3	2.5	2.9	3.5
5	2.3	2.8	3.1	3.5	4.2
10	2.8	3.4	3.8	4.3	5.1
15	3.1	3.7	4.1	4.7	5.6
20	3.3	4.0	4.4	5.1	6.0
25	3.5	4.2	4.6	5.3	6.3
30	3.7	4.4	4.8	5.6	6.6
40	3.9	4.6	5.1	5.9	7.0
50	4.0	4.7	5.3	6.1	7.2

TABLE 32 (Cont'd)

South Central Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.8	2.1	2.3	2.6	3.0
3	2.1	2.5	2.8	3.2	3.7
5	2.6	3.1	3.3	3.8	4.5
10	3.2	3.8	4.2	4.7	5.6
15	3.4	4.2	4.6	5.2	6.1
20	3.8	4.5	4.9	5.6	6.6
25	4.0	4.7	5.3	5.9	7.0
30	4.2	4.9	5.5	6.2	7.3
40	4.4	5.2	5.8	6.5	7.7
50	4.6	5.4	6.0	6.8	8.1

Southeast Section

Recurrence Interval (Yrs.)	Depth (In.) for Given Period (Days) 1	2	3	5	10
2	1.9	2.2	2.5	2.8	3.3
3	2.3	2.7	3.0	3.4	4.0
5	2.8	3.3	3.6	4.1	4.8
10	3.4	4.0	4.4	5.0	5.9
15	3.7	4.4	4.8	5.5	6.4
20	4.0	4.7	5.2	5.9	6.9
25	4.2	4.9	5.5	6.2	7.3
30	4.4	5.1	5.7	6.4	7.6
40	4.6	5.4	6.0	6.8	8.0
50	4.8	5.6	6.3	7.1	8.4

As stated previously, calendar-day values for the annual series in Tables 23 to 32 may be converted to partial duration, maximum period values by the transformation factors given in Tables 20 and 21. The conversion process is illustrated by the following example:

> Assume that it is desired to convert the 1-day, 5-yearrecurrence interval value for the Northwest Section to its equivalent partial duration, maximum 24-hour value. Reference to Table 23 shows the above recurrence value to be 3.2 inches. First convert from calendar-day to maximum 24-hour amount by multiplying 3.2 inches by 1.13, the conversion factor for 1-day precipitation in Table 21. This results in a value of 3.62 inches. Then multiply 3.62 inches by 1.05, the factor for converting from annual to partial duration series given in Table 20 for a 5-year recurrence interval of 1-day precipitation. This multiplication gives a value of 3.8 inches, the desired partial duration, maximum 24-hour value for the 5-year recurrence interval.

TABLE 33

RATIO OF PRECIPITATION FOR 10, 5, 3, AND 2 DAYS TO 1 DAY

Storm Period (Days)	Ratio for Given Storm Period (Days)				
	Annual	Winter	Spring	Summer	Fall
			Northwest Section		
2	1.19	1.21	1.22	1.21	1.17
3	1.32	1.31	1.40	1.33	1.29
5	1.50	1.50	1.62	1.52	1.47
10	1.80	1.73	1.98	1.79	1.72
			North Central Section		
2	1.20	1.18	1.23	1.20	1.19
3	1.33	1.30	1.40	1.32	1.32
5	1.52	1.47	1.61	1.52	1.53
10	1.82	1.73	1.97	1.82	1.81
			South Central Section		
2	1.21	1.19	1.25	1.21	1.17
3	1.34	1.33	1.39	1.34	1.29
5	1.54	1.49	1.63	1.52	1.46
10	1.86	1.76	2.01	1.82	1.72
			Southeast Section		
2	1.20	1.23	1.22	1.21	1.17
3	1.33	1.35	1.36	1.34	1.31
5	1.52	1.57	1.57	1.51	1.48
10	1.82	1.90	1.90	1.81	1.74

Table 33 is presented to illustrate relations between the several storm periods. Ratios of precipitation for 10, 5, 3, 2 days to 1-day are given on an annual and seasonal basis for each section. The ratios were determined from the frequency relations presented in Tables 23 and 29 to 32. The ratios reach a maximum in spring in all except the Southeast Section, where winter and spring are equal.

The rate of increase of precipitation with increasing recurrence interval is illustrated on an annual and seasonal basis in Table 34, using frequency relations developed for each section. In this table, the ratios of precipitation for 50, 25, 10, and 5 years to 2-year return period precipitation are shown. With respect to the four seasons, the ratios reach a maximum for summer in all sections, indicating the year-to-year variability in the magnitude of storm precipitation is greatest in this season. Combining all sections, spring is the season of lowest ratios or minimum variability with respect to time. All sections have nearly identical ratios in fall, indicating equal variability throughout the state. In extreme southern Illinois, where the heavy storms are

TABLE 34

RATIO OF PRECIPITATION FOR 50, 25, 1Q, AND 5 YEARS TO 2 YEARS
FOR STORM PERIODS OF 1 TO 10 DAYS

Recurrence Interval (Yrs.)	Ratio for Given Recurrence (Yrs.)				
	Annual	Winter	Spring	Summer	Fall
		Northwest Section			
50	2.7	2.3	2.3	3.2	2.6
25	2.2	2.1	2.1	2.6	2.3
10	1.7	1.7	1.7	1.9	1.8
5	1.4	1.4	1.4	1.5	1.5
		North Central Section			
50	2.4	2.5	2.3	3.2	2.5
25	2.0	2.2	2.1	2.6	2.2
10	1.6	1.8	1.7	1.9	1.8
5	1.3	1.5	1.4	1.5	1.5
		South Central Section			
50	2.5	2.6	2.5	3.6	2.5
25	2.1	2.3	2.1	2.8	2.2
10	1.6	1.8	1.8	2.0	1.8
5	1.3	1.5	1.5	1.5	1.5
		Southeast Section			
50	2.3	2.8	2.5	3.6	2.5
25	2.0	2.4	2.2	2.9	2.2
10	1.6	1.9	1.8	2.0	1.8
5	1.3	1.5	1.5	1.5	1.5

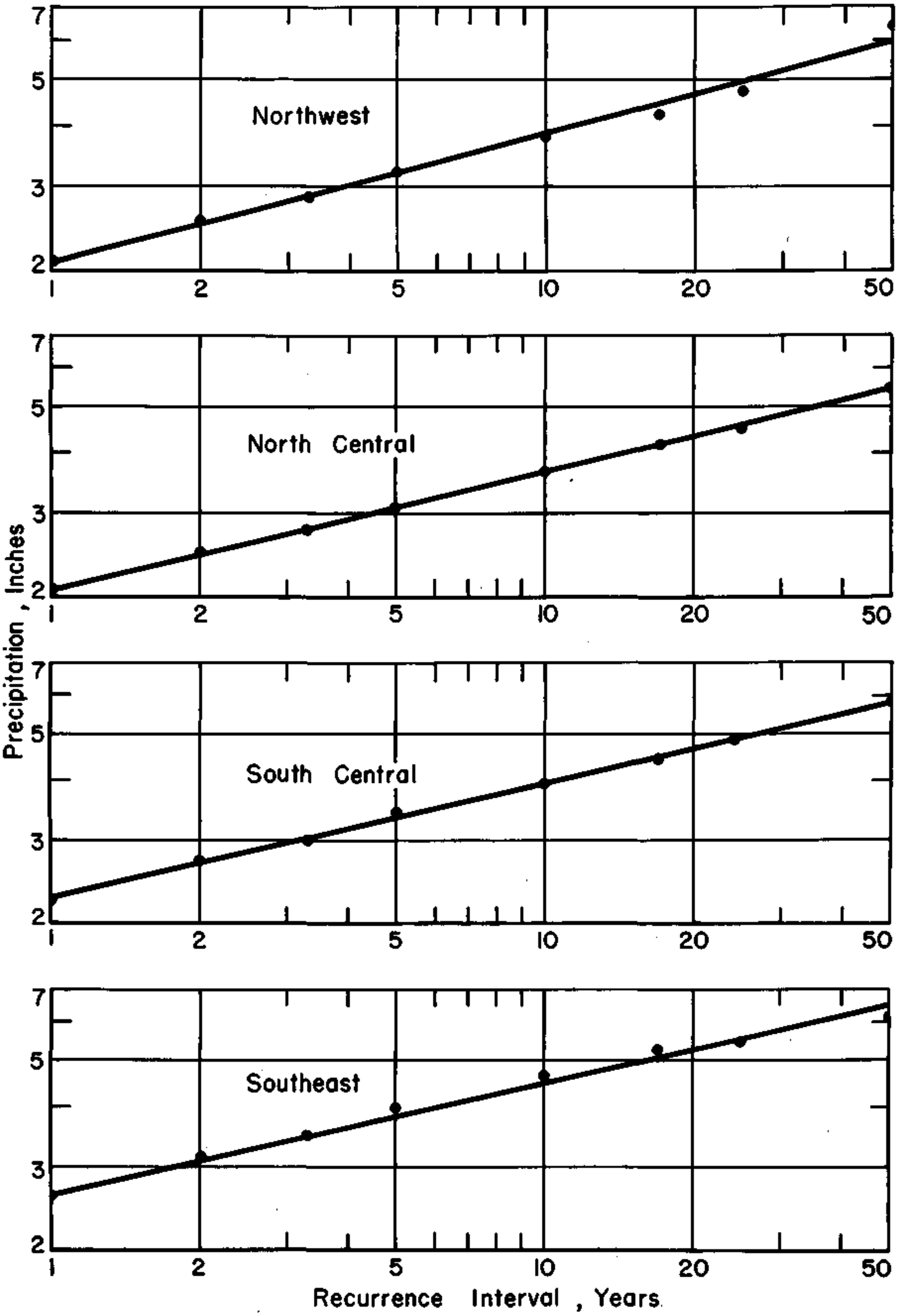

FIGURE 14 PARTIAL DURATION ANALYSIS OF 1-DAY PRECIPITATION FREQUENCIES

distributed more uniformly through the year, the annual ratios are least. Conversely, the highest annual ratios occur in the Northwest Section where most of the heavy storms occur from late spring to early fall.

The magnitude of the ratios for long recurrence intervals is considerably affected by the statistical distribution used in deriving the frequency relations. For example, if the log-normal distribution is used instead of the Frechet method, the 50-year to 2-year annual ratios become 2.0, 1.9, 2.0, and 1.9, respectively, for the Northwest, North Central, South Central, and Southeast Sections. The log-normal 50-year values for 1-day precipitation are 4.9, 4.5, 5.1, and 5.5 inches, respectively, for the above four sections compared to 6.4, 5.4, 6.2, and 6.5 inches obtained from the Frechet analysis presented in Table 23.

Frequencies for 1 Day from Partial Duration Data

A frequency distribution of 1-day precipitation was determined from data for the partial duration series. Analysis of the partial duration series was accomplished with daily precipitation data available on IBM punch cards. Data for 51 stations were available on punch cards at the time of the analysis. These included stations with records spanning a period of 45 to 55 years during the period 1901-1955. The same analysis technique was followed as with the annual maxima data. That is, the state was divided into the same four sections shown in Figure 1 and average frequency relations determined for each section. There were 14, 16, 12, and 9 stations, respectively, available for determining average relations in the Northwest, North Central, South Central, and Southeast Sections.

Frequency curves for each section obtained by applying the partial duration series to annual data are shown in Figure 14. The curves were fitted by graphical analysis. An excellent fit was obtained on a log-log scale, as shown by the position of the plotting points in Figure 14. No further attempt was made to test other curve fitting methods, considering the excellent fit obtained by graphical analysis and the experience gained in the detailed analysis of annual maxima data.

Excellent agreement was obtained between the relations developed from the partial duration series and those determined from annual maxima data for a somewhat shorter and different period of time. Comparison of results from the two methods is presented in Table 35 for selected recurrence intervals. Despite the use of non-equivalent observational periods, the empirical relation between partial duration and annual series frequencies presented in Table 20 is indicated satisfactorily except for the North Central Section.

When the annual series values for periods of 1 to 10 days in Table 23 are converted to partial duration series values by applying the appropriate transformation factors from Table 20, an excellent straight-line fit is also obtained on a log-log plotting scale. However, seasonal data do not fit the log-log relation as satisfactorily when converted to partial duration series values.

The results of the foregoing investigation suggest that simple graphical analysis methods are generally satisfactory for the analysis of frequency data.

Relation Between Annual Precipitation and Storm Frequencies

Investigation was made of the relation between annual precipitation and precipitation frequencies for storm periods of 1 to 10 days. This investigation was made to evaluate further the relations between small-scale and large-scale climatic events, which were indicated in a previous section dealing with the monthly and seasonal distribution of annual maxima.

Annual mean precipitation data for the 50-year period, 1906-55, were used in the analysis. The data for this period were available from an unpublished Illinois State Water Survey study, and the period is of sufficient length to provide a reliable estimate of the population means. The annual precipitation means are presented in Figure 15. The storm period frequencies derived for annual maxima applying the Frechet method (Table 23) were related to the annual means.

First, a station comparison was made between annual mean precipitation and the means of the annual maxima for storm periods of 1 to 10 days. The storm means were expressed as a percentage of the annual means in this phase of the study. Results of this analysis indicated a close association in the four climatic sections. For 1-day precipitation in the Northwest Section, percentages ranged from 7.3 to 8.1 with an average of 7.7. Similarly, the North Central Section had stations ranging from 6.5 to 7.6 with an average of *6.9*. In the South Central Section the average was 6.8 with a range of 6.4 to 7.2, while the Southeast Section had an average of *6.9* with a range from 6.4 to 7.7. Combining all four sections, correlation coefficients of 0.74, 0.83, and 0.92 were obtained between mean annual and 1-day, 5-day, and 10-day mean precipitation, respectively.

TABLE 35

COMPARISON OF PARTIAL DURATION AND ANNUAL MAXIMA ANALYSES FOR 1-DAY PRECIPITATION

Recurrence Interval (Yrs.)	Precipitation (In.) for Given Method Partial Duration	Annual Maxima	Difference (In.)
	Northwest Section		
40	5.7	5.9	0.2
20	4.7	4.8	0.1
10	3.9	3.9	0.0
5	3.2	3.2	0.0
2	2.5	2.3	0.2
	North Central Section		
40	5.2	5.1	0.1
20	4.3	4.3	0.0
10	3.6	3.6	0.0
5	3.1	3.0	0.1
2	2.1	2.2	0.1
	South Central Section		
40	5.5	5.8	0.3
20	4.7	4.9	0.2
10	3.9	4.0	0.1
5	3.3	3.3	0.0
2	2.7	2.4	0.3
	Southeast Section		
40	6.2	6.2	0.0
20	5.3	5.3	0.0
10	4.5	4.4	0.1
5	3.8	3.7	0.1
2	3.1	2.8	0.3

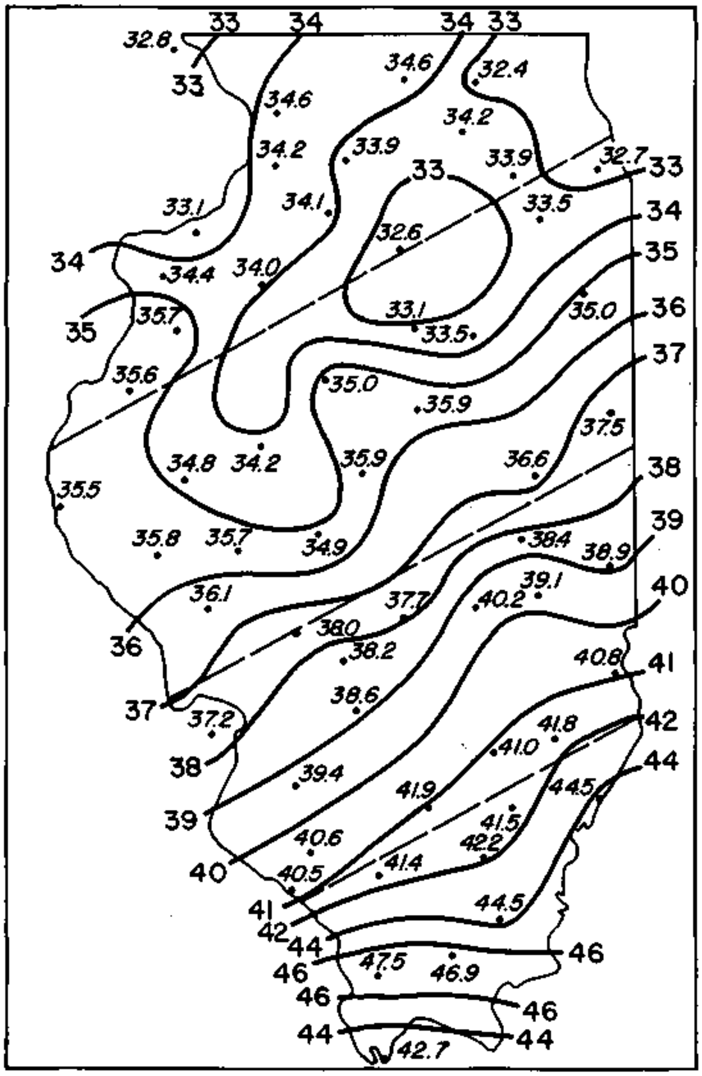

FIGURE 15 MEAN ANNUAL PRECIPITATION, 1906-55

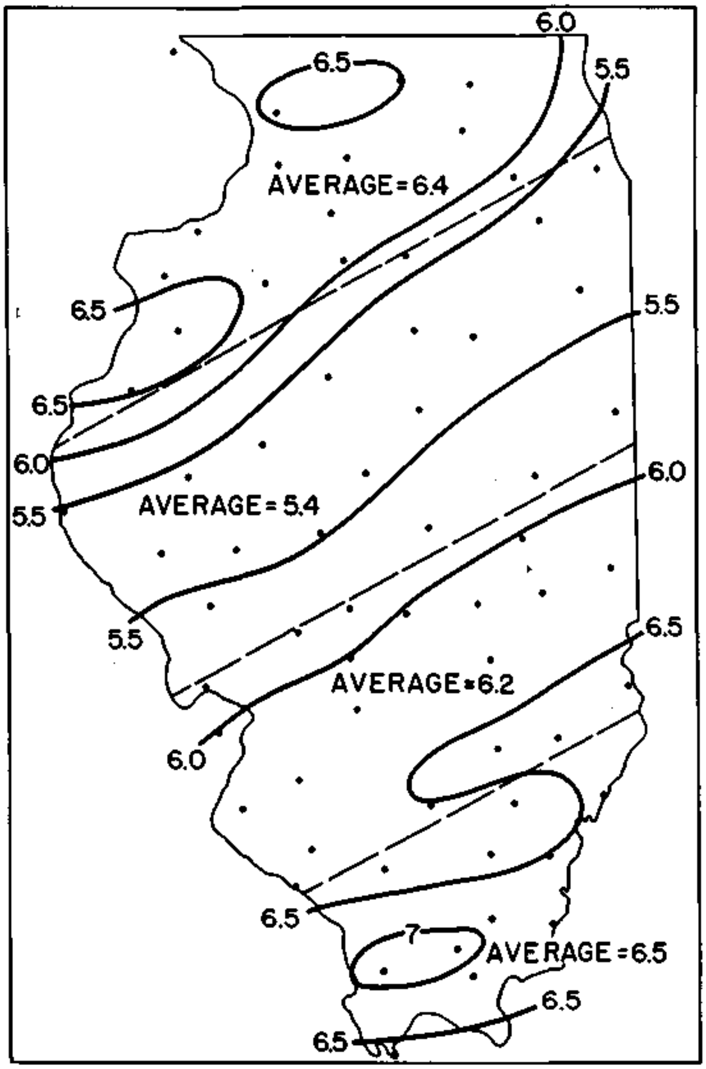

FIGURE 16 50.YEAR RECURRENCE OF 1-DAY PRECIPITATION BASED ON RELATION BETWEEN ANNUAL AND STORM PRECIPITATION

Next, the relation between the average annual precipitation and storm precipitation for various recurrence intervals and storm periods was determined for each of the four climatic sections. Results of this analysis are shown in Table 36, where the storm period precipitation values have been expressed in terms of percent of annual mean precipitation at selected recurrence intervals for each section. Except for the Northwest Section, the relation between annual and storm precipitation is nearly uniform throughout the state.

The data from Table .36 and Figure 15 provide another method for obtaining isohyetal patterns. By multiplying the station annual means by the percentage value for the corresponding section and recurrence interval in Table 36, numerous points can be obtained for establishing an isohyetal map. This procedure has been illustrated for a 50-year recurrence of 1-day precipitation in Figure 16. Average sectional values from Table 36, which are also shown in Figure 16, indicate a relatively small range of values about the mean in each section. Figure 16 may be compared with Figures 2 and 3 to indicate the differences between isohyetal patterns derived from several techniques using the same statistical distribution (Frechet).

In Figure 16, the isohyetal pattern is based upon average storm frequencies for climatic sections and station annual mean precipitation. The isohyetal pattern of Figure 2 was obtained from the same sectional storm frequencies and the storm precipitation means for individual stations. Figure 3 was derived entirely from storm frequency relations for individual stations. For those who prefer an isohyetal presentation, the authors recommend the relatively simple method employed in this section, using an-

TABLE 36

RELATION BETWEEN STORM FREQUENCIES AND ANNUAL MEAN PRECIPITATION USING ANNUAL MAXIMA AND LOG GUMBEL METHOD

Storm Recurrence (Yrs.)	Percent of Annual Mean for Given Storm Period (Days) 1	2	3	5	10
			Northwest Section		
2	6.8	8.2	9.1	10.2	12.3
5	9.2	11.0	12.3	14.0	16.6
10	11.4	13.7	15.2	17.3	21.5
20	14.0	16.8	18.7	21.0	25.2
30	16.1	19.0	21.0	23.7	28.4
40	16.9	20.3	22.5	25.5	30.5
50	18.7	21.7	24.3	27.3	33.1
100	21.8	26.2	28.9	32.8	38.7
			North Central Section		
2	6.4	7.7	8.5	9.6	11.7
5	8.4	10.1	11.2	12.6	15.4
10	10.2	12.2	13.6	15.3	18.8
20	12.3	14.8	16.4	18.5	22.5
30	13.7	16.4	18.3	20.6	24.8
40	14.6	17.5	19.5	21.9	26.8
50	15.4	18.5	20.5	23.1	27.9
100	18.2	21.8	24.2	27.2	33.4
			South Central Section		
2	6.2	7.4	8.3	9.3	11.5
5	8.3	10.0	11.1	12.5	15.7
10	10.1	12.1	13.5	15.2	19.3
20	12.4	14.9	16.5	18.6	23.5
30	13.8	16.5	18.4	20.7	26.0
40	14.8	17.6	19.7	22.2	27.4
50	15.7	18.9	20.9	23.5	29.5
100	18.8	22.6	25.1	28.2	35.0
			Southeast Section		
2	6.4	7.7	8.5	9.6	11.7
5	8.4	10.1	11.2	12.6	15.3
10	10.1	12.1	13.5	15.2	18.4
20	12.0	14.4	16.0	18.0	22.0
30	13.3	16.0	17.7	19.9	24.3
40	14.2	17.1	18.9	21.3	25.9
50	14.8	17.8	19.7	22.2	26.6
100	17.7	21.2	23.6	26.6	32.6

nual station means from Figure 15 with the percentages of Table 36. For reasons given previously, the authors prefer use of sectional rather than station data for derivation of storm frequencies. The relative variability of annual precipitation is considerably less than that for storm precipitation. Consequently, the 50-year annual means for stations should provide more accurate estimates of the population annual means' than do the 40-year station storm means used for estimating the population storm means from which Figure 2 is derived. There is also indication in Figure 16 that the recommended method is more satisfactory climatologically. Precipitation centers shown in extreme northwestern and southern Illinois in Figure 16 coincide with the Rock River Hills and the Shawnee Hills, the most prominent hill regions in the state, which may have some augmenting effect on storm precipitation.

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