Contract 38

INVESTIGATION OF WATER DROPLET COALESCENCE

by

C. H. Hendricks

R. G. Semonin

Final Report

Signal Corps Grant No. DA-SIG-36-039-62-G19

ARPA Order No. 265-62

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Amount of Grant: $35,000

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26 January 1962 to 26 October 1962

CHARGED PARTICLE RESEARCH LABORATORY

ILLINOIS STATE WATER SURVEY AND

DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF ILLINOIS

URBANA, ILLINOIS

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ABSTRACT

To obtain quantitative results on the time it takes precipitation of droplets to develop within a cloud, the droplet collision and coalescence processes are being investigated theoretically and experimentally under static and dynamic conditions. The factors which aid and hinder the coalescence of, two water droplets are being studied in a humidity and temperature controlled, field-free space. The electric field is also known to effect the collision and coalescence processes. The work includes the effects of applied electric fields on: (a) collision efficiency, (b) deformation of droplet surfaces when two droplets approach each other, and (c) the phenomenon of charge transfer between two droplets in proximity.

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1. INTRODUCTION

Two major factors involved in the growth of raindrops are the collision and coalescence of cloud droplets., The rate of growth of cloud droplets from 1 micron to 15 microns in radius can be accounted for by the process of diffusion. For these small cloud droplets to reach raindrop size by the condensation process requires a growing period of approximately 24 hours. This process in conjunction with the coalescence process can account for light, steady precipitation, but not the precipitation that occurs within several minutes from non-freezing clouds. It has been suggested that the intense electric fields within clouds play an important role in the growth of raindrops via the coalescence process. It has been shown by various investigators that the collision efficiency is definitely increased if there is an electrical attraction between the cloud droplets. Thus, the research is directed to the examination of these factors in greater detail.

It has been experimentally demonstrated that in field-free space not every collision results in coalescence of the droplet pair. Thus, the droplet coalescence experiment examines the factors which aid and hinder the coalescence process under static and dynamic conditions. The literature[2] has indicated that the saturation of the atmosphere surrounding the droplet with the vapor of the same liquid is favorable for their coalescence and a deficiency of the vapors hinders their coalescence. It is normally assumed that the air film trapped between two colliding droplets prevents their coalescence. However, the coalescence of two droplets is strongly dependent on the ambient humidity. To elucidate the effect of humidity, experiments under both static and dynamic conditions are being carried out.

Other factors influencing the coalescence of droplets are the conditions of their surfaces and electrical forces between them. Detailed calculations are now being carried out to determine the effects of electric fields on droplet surfaces, on increase in collision efficiencies, and on coalescence.

2. EXPERIMENTAL STUDIES

A. Coalescence Experiment

A major factor in the formation of rain is the coalescence of cloud droplets. The purpose of this experiment was to examine factors which prevent or aid the coalescence process when the drop surfaces are in close proximity.

At present, the literature (2,3,4) indicates a trapped film of air, sometimes referred to as the air-vapor gap or polymolecular film, is the main reason liquid drops do not coalesce immediately upon contact. The existence of a vapor gap between two liquid drops has been experimentally demonstrated by Prokhorov[2]. The vapor gap exists only when there is a humidity deficit. Compared with water, the liquids examined by Prokhorov were highly volatile and his results cannot be applied directly to water drops. In his examination of water, he indicates the existence of the vapor gap was not so decisive. Thus, the specific aim in this coalescence experiment was to determine the existence of the vapor, gap between two water drops. If the experiment shows a vapor gap preventing two stationary drops from coalescing, then a future problem, described in the collision experiment, is to determine whether two moving drops can be made to bounce in a region with a humidity deficit. If with a humidity deficit the drops bounce, good confirmation of the theory is indicated. However, if the drops consistently coalesce, then the theory of a vapor gap preventing coalescence is probably not valid.

As has already been mentioned in previous reports, Linton and Sutherland[3] have shown that the coalescence of two drops (not water) is delayed by the air film trapped between opposing surfaces. The delay time for the coalescence of two water droplets will be measured in the collision experiment.

The following work has been accomplished in measuring the vapor gap between two water drops:

a) Water drops supported by glass fibers (see Figure 1) and then transported towards each other appeared to coalesce immediately upon contact. Each of the two glass fibers were mounted in a brass clamp,

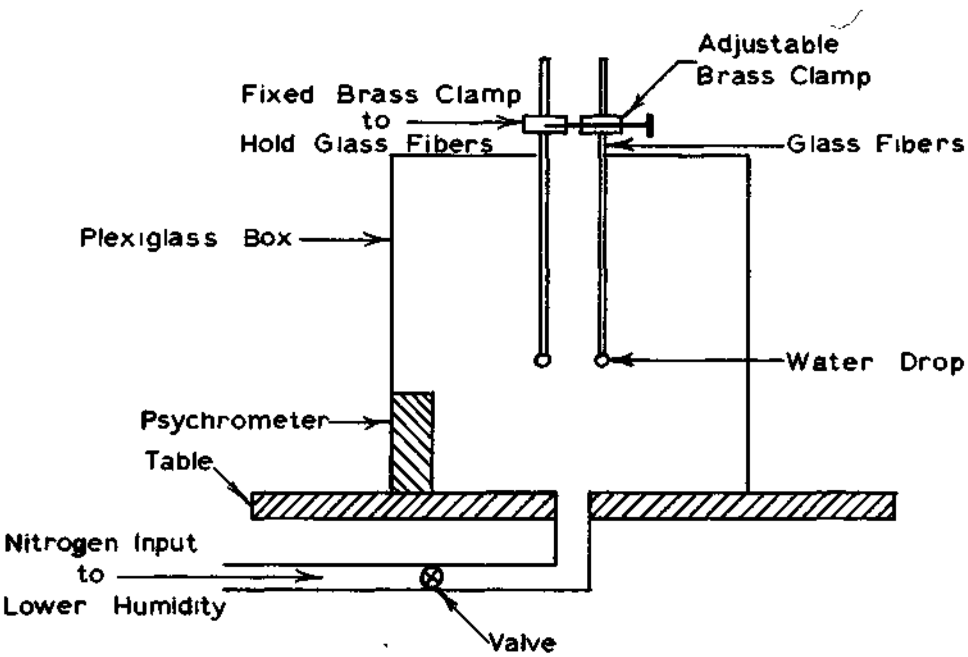

Figure 1

Initial Experiment To Observe Drop Coalescence which was adjustable in the horizontal direction so the drop separation could be varied. The drops were viewed through a ten power telescope and illuminated with diffused light. With this arrangement the drops could be brought fairly close together, but the vibrations in the horizontal separator prevented making any micro-horizontal adjustments. These vibrations would cause the glass fibers to vibrate, which in turn caused the drops to vibrate.

A delay in the coalescence was expected with a humidity deficit. But, varying the humidity between 15 and 50 percent had no visual effect on drop coalescence. The vibrations in the table and the horizontal separator prevented bringing the drops close enough to observe the delay time.

The following apparatus has been constructed to minimize the vibrations.

b) A concrete table mounted on a one-inch thick cork pad has been built to minimize the floor and other extraneous vibrations. The complete apparatus is sketched in Figure 2.

c) The mechanism to form and house the drops is shown in detail in Figure 3. This method of bringing the drops together eliminates vibrations transmitted to the drops by the hands/

d) The optical equipment has been set up so the interference bands can be photographed and measured. From the geometry and the order of the interference bands, the shape and thickness of the vapor gap can be determined. Prokhorov determined the thickness of the gap as follows: Consider the diagram shown in Figure 4a of two drop surface at some separation, t. Suppose that the interference pattern is observed. From the properties of thin films we know that the thickness of the air film between the two drop surfaces is given by

$$t = (m + 1/2)\frac{\lambda_1}{2n}$$

where m is an interger, λ_1 is the wavelength of light, and n is the index

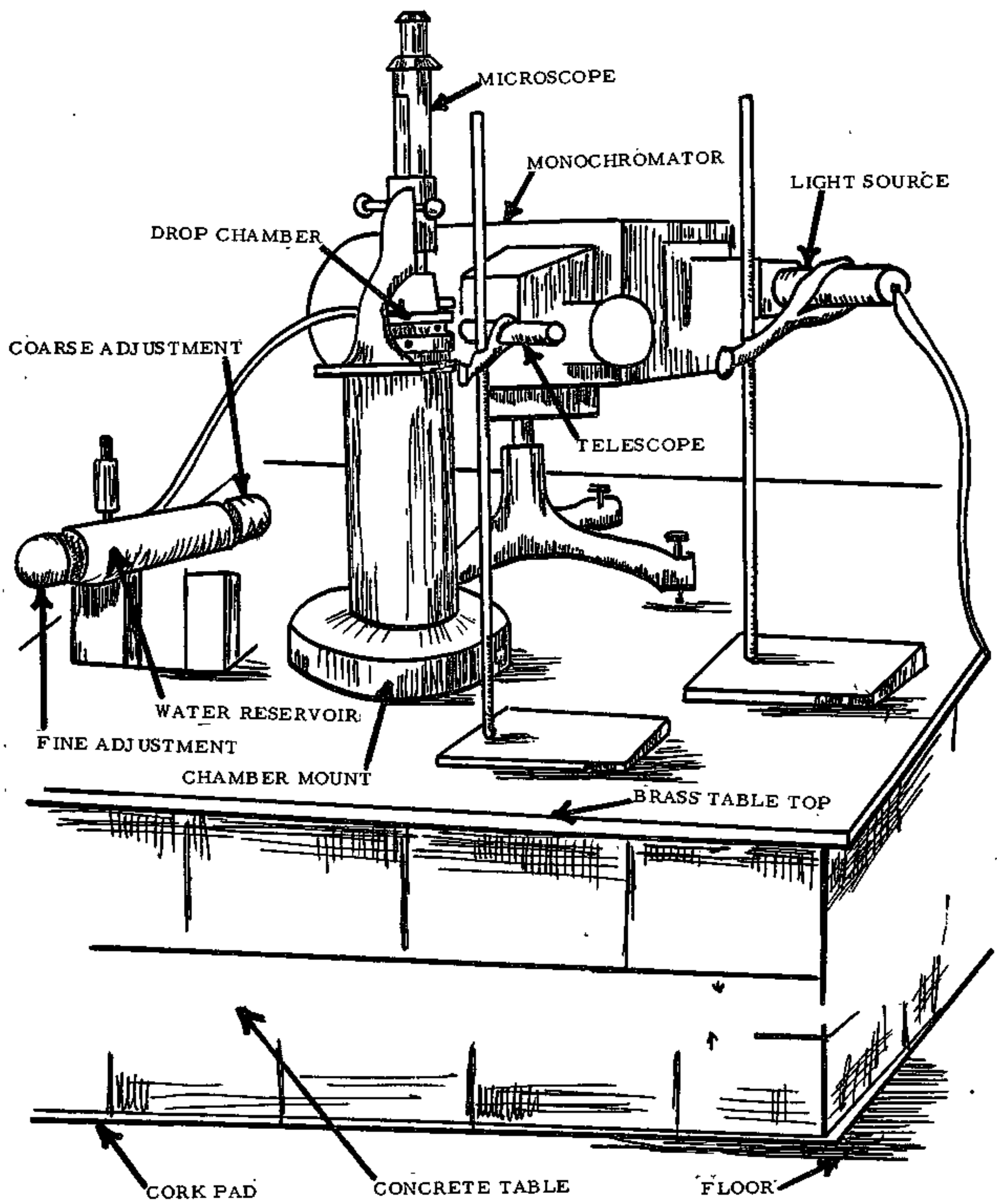

Figure 2.

General Setup of Coalescence Experiment

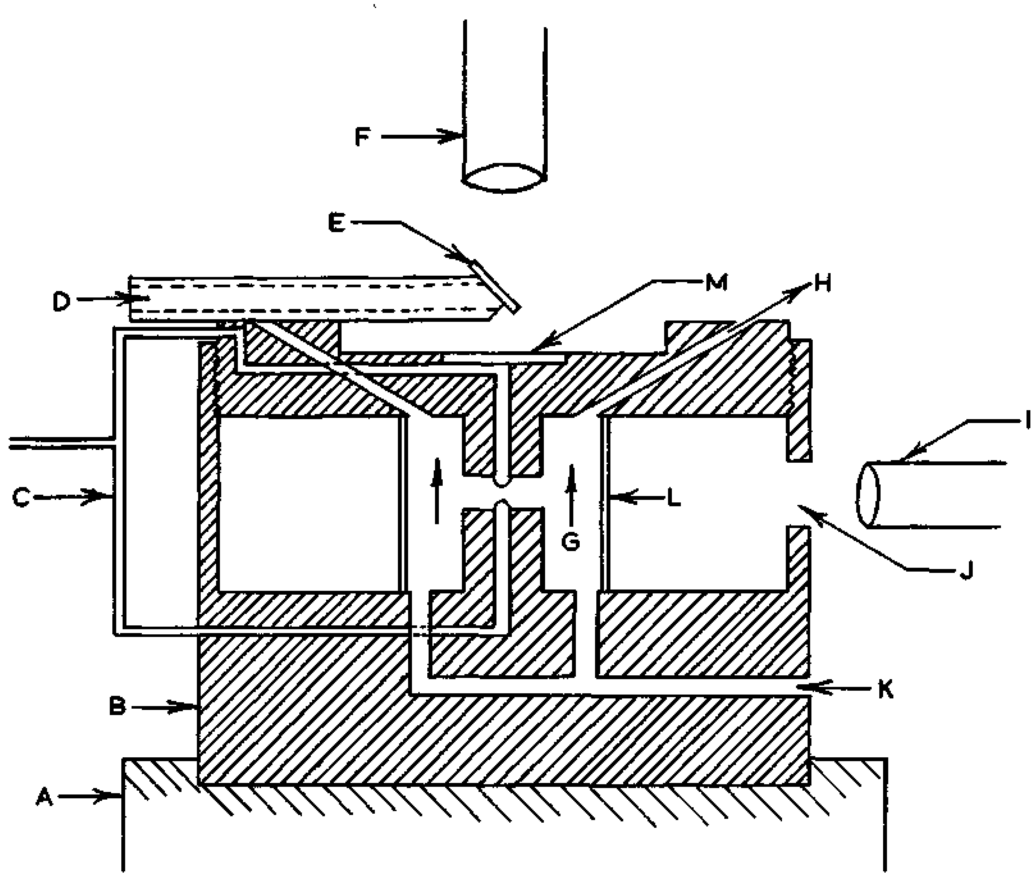

Figure 3

Cross Sectional View of Drop Chamber

A - Steel tube on which drop chamber is mounted
B - Drop chamber
C - Water input line
D - Light from the monochromator
E - Half silver mirror
F - Low power microscope to observe interference bands
G - Air flow through the chamber
H - Air output from chamber
I - Low power telescope to view drops
J - Window
K - Nitrogen input
L - Glass cylinder
M - Glass slide

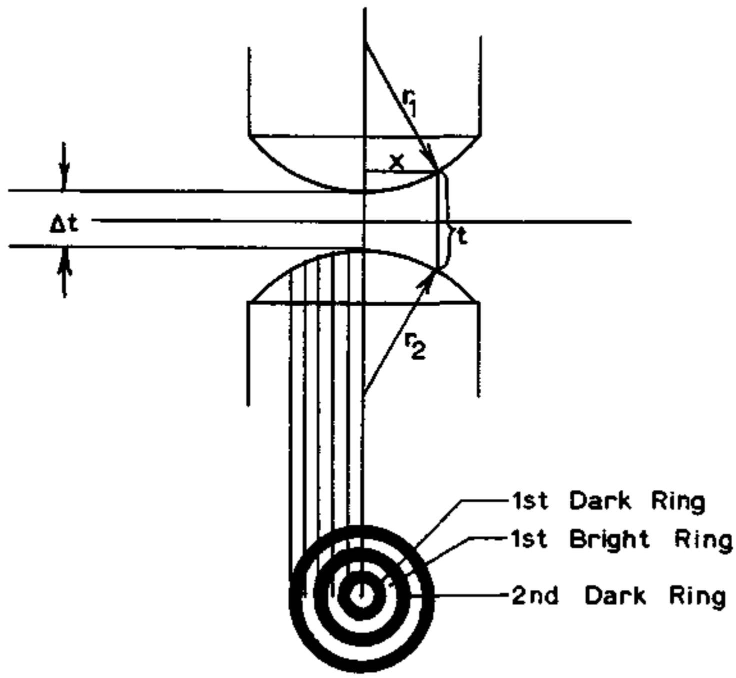

Figure 4a

Geometry and Interference Bands Between Droplet Surfaces

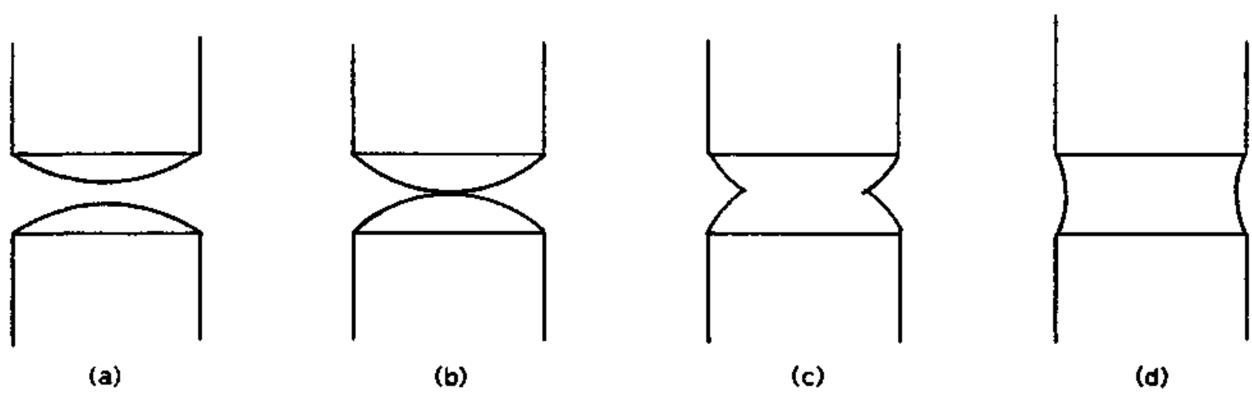

Figure 4b

Side View of Droplet Surfaces in Close Proximity of refraction. Since the radius of a ring is proportional to λ, a decrease in wavelength will decrease the radius of the rings. So by decreasing the wavelength the first bright ring can be brought into the center of the interference pattern. This change is observed through the microscope and the wavelength for which the first bright ring moves to the center is λ_2. which is determined from the monochromator. In this change, m changes by one. Since the thickness of the air film remains constant we must have

$$t = (m + 1/2)\frac{\lambda_1}{2n} = (m + 1/2 + 1)\frac{\lambda_2}{2n}$$

or

$$t = (m + 1/2)\frac{\lambda_1}{2n} = (m + \frac{3}{2})\frac{\lambda_2}{2n}$$

If we eliminate m from the above equations, we obtain

$$t = \frac{\lambda_1 \lambda_2}{2n(\lambda_2 - \lambda_1)}$$

which gives the thickness in terms of wavelength. However, this method is only accurate to an integral number of half wavelengths because varying the air film thickness by any whole number of half wavelengths does not change the position of the rings. Thus, the method used by Prokhorov does not give the absolute thickness of the film.

The following procedure eliminates the above ambiguity in determining the air film thickness. The monochromator is set to the shortest visible wavelength, and the separation between the drop surfaces then is reduced and adjusted until a bright spot appears in the center of the interference pattern. With a bright spot at the center, the thickness of the air film at the center is given by $t = (m + 1/2)\frac{\lambda}{2n}$ where m in unknown. If m = 0, an increase in wavelength of a quarter of a wavelength will bring a dark ring into the center, and any further increase in wavelength will not change the interference pattern because now the air film is thin relative to the wavelength. However, if m had not been zero, a further increase in wavelength would have changed the interference pattern by bringing in another bright ring. Thus, to determine the absolute thickness, the drop separation is reduced until the condition m = 0 is fulfilled. This is one way to determine the absolute separation and at best the separation can be determined to only a quarter of a wavelength of visible light.

The separation, t, can also be obtained in terms of the geometry. In Figure 4a, r_1 and r_2 are the radii of curvature of the drops. The separation of the two curved surfaces can therefore be written approximately,

$$t = \frac{x^2}{2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) + \Delta t$$

where the approximation made is $x \ll r_1$ and r_2. In the case of equal drops we have

$$t = \frac{x^2}{r} \qquad \text{where } r_1 = r_2 = r$$

The following procedure was used to bring the drops together. First, the humidity was lowered in the drop chamber by pumping in nitrogen. Then, the drops were formed with the coarse adjustment. With a fast flow of dry nitrogen into the chamber, the drops could be "rammed" together without coalescing. With the drops in this position the interference pattern was very distorted. The distortion was produced by the rapid nitrogen flow which caused the drop surfaces to vibrate and distort the rings. As soon as the flow was turned off the drops coalesced. This indicates that the rapid flow of nitrogen around the drops prevents them from coalescing. The above distortion in the interference pattern was removed by simultaneously reducing the flow and increasing the separation between the drop surfaces. Although a perfect interference pattern could be observed, the pattern changed rapidly making it impossible to measure the film thickness. This is a mechanical problem due to the lack of a sufficiently fine adjustment at the water reservoir. This and a few other minor problems must be solved before a measurement of the thickness can be made.

A delay in the coalescence appears to occur when there is a humidity deficit. The drop surfaces appear to flatten after they touch, and can be maintained in this position for some time. Without a humidity deficit, the drops coalesce upon contact. Figure 4b shows side views of the drop surfaces as seen through the telescope mounted on the side of the drop chamber. In view (a) the drop surfaces have not touched; in (b) the surfaces are just touching; and in (c) the surfaces are touching, but area of contact is wider than in (b), and at the point of contact the two surfaces cannot be distiaguished. View (d) shows the water surface after the drops have coalesced.

B. Charge Separation Experiment

Many of the recent investigations on the formation of clouds and rain have indicated that the electric fields play an important role in this formation. Before any cloud formation begins, the electric field "in the region is usually quite low but it builds up very rapidly along with the growing cloud. This seems to imply that a regenerative process or processes are involved in the redistribution of the charge that must be associated with the electric field buildup.

Probably several phenomena take place in the redistribution of the charge in a cloud, but one interesting proposal for charging of neutral particles was given by Sartor[5] . In this paper he described an experiment in which the observed events were explained by a charge transfer between two water droplets on the tip of two glass fibers in the presence of an electric field. For a close separation of the two water surfaces, the droplets moved together as the electric field was increased, then separated suddenly. With a further increase in the electric field the droplets again move together. Since the electric field is enhanced between the surfaces which are very close together, a breakdown of the air with a charge transfer seems reasonable. Therefore, an investigation was initiated to examine this possible phenomenon for charge separation.

The first approach used to observe and measure possible charge transfer between two water drops employed two falling water drops. One would fall on and slide down a teflon-insulated wire. Another larger drop was allowed to fall an instant later and pass the first drop which was still on the wire. The friction of the teflon and the lower terminal velocity of the smaller drop allowed the larger drop to travel at the slightly higher velocity required for passing. Two large metal plates were placed on each side of the wire so an electric field could be applied. The timing of the drops formed was controlled by using a solenoid to pulse the pressure of the water supply systems. This forced the formation of the drops and their separation from a needle tip. The separation between the surfaces of the passing drops was varied by changing the distance between the needle tips. Figure 5 is a sketch of the experimental setup showing the position of the various parts.

An electrometer was connected to the teflon-insulated wire. If the charge on the drop changed as it slid down the teflon it would induce a charge on the inner conductor and show as a deflection on an electrometer. Therefore, if any charge was transferred to this drop as the second drop passed, the electrometer would give an indication, and the amount of the deflection of the meter could be calibrated to give the magnitude of the charge transferred.

This system did not prove to be successful. At first it was found that it was impossible to form water drops from the end of a needle, made from either a metal or a dielectric, without the drops being charged. This charge gave a deflection of the electrometer and masked any deflection caused by true charge transfer. Then a cage containing a polonium source was placed under the needles so the drops fell through an ionized region. As a drop, which was charged, passed through the ionized particles, most of the charge on the drop was carried away leaving the drop almost neutral. However, it was found that uncharged drops still produced a deflection on the electrometer. This implied that charging occurred as the neutral drops came in contact with the teflon or as the surface of the drop was changed. The neutrality of the drops was determined by adjusting the drops so they would

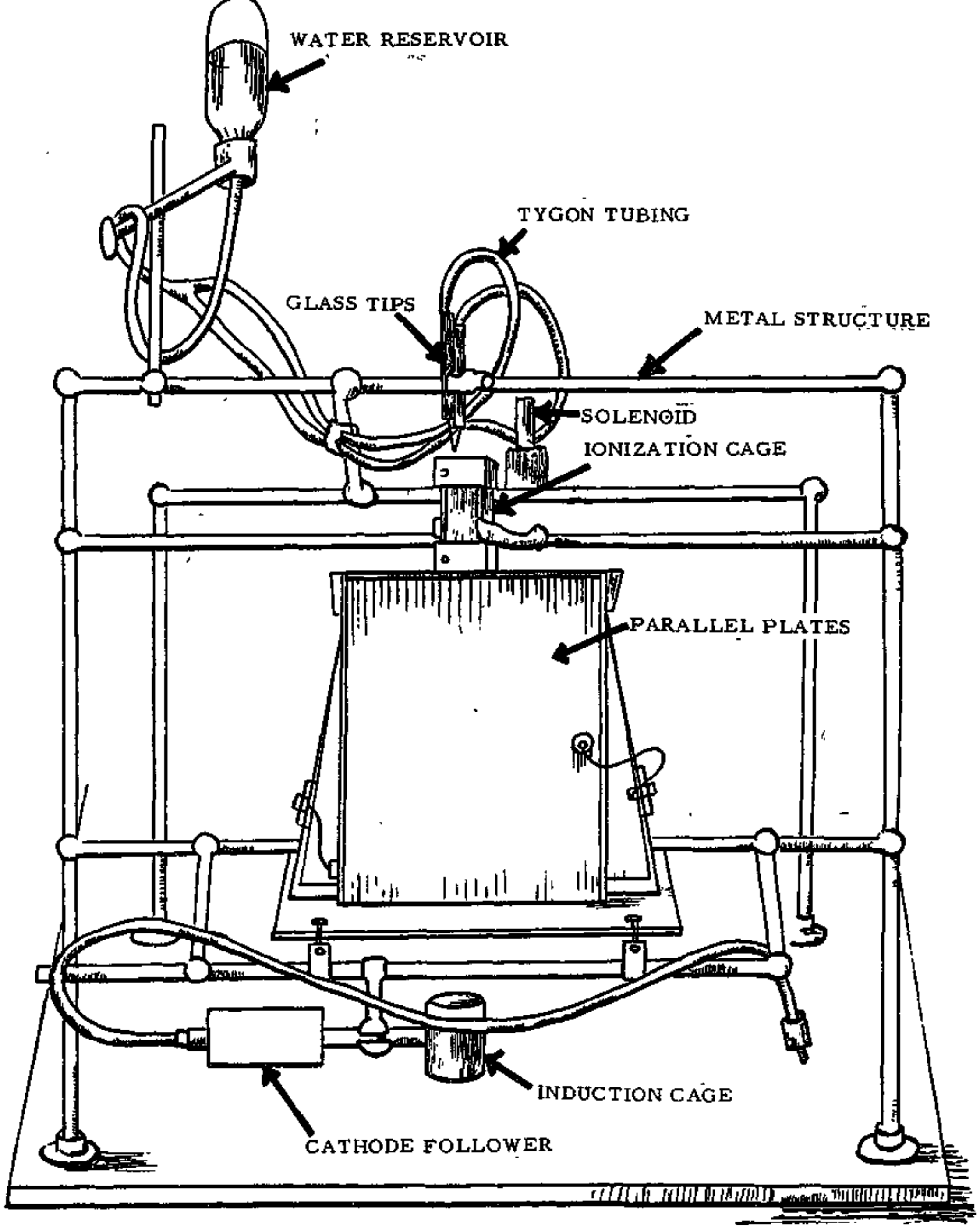

Figure 5

Initial Charge Transfer Apparatus

pass by but not come in contact with the teflon insulation. Without passing through the polonium cage, the drop would give an electrometer deflection but after passing through the cage, only a very slight deflection could be detected. Then by moving the needle so the drop contacted the teflon, a sizeable deflection was observed.

To eliminate this disturbance of the water drops, an induction cage was made from two concentric metallic cylinders with the outer cylinder at ground potential. The inner cylinder was supported and insulated from the outer cylinder by a glass feed-through. This gave an impedance between the two cylinders of about 10^{14} ohms. The inner cylinder was connected to a high input impedance electrometer tube used as a cathode follower. The output of the cathode follower was displayed on an oscilloscope to measure the output voltage pulses. With such a high impedance between two cylinders, the cage functioned as a very good capacitor. When a charge passed through the center, an approximately equal charge, Q, was induced on the inner cylinder. Therefore, since the cage acts as a capacitor with known capacitance, C, the output voltage pulse was $V = Q/C$. Consequently, the charge passing through the cage was given by $Q = CV$. The obvious advantage of this device was that the particle being measured was not disturbed, A disadvantage was that the induction cage being used has a lower limit for measuring charge of about 10^{-14} coulomb. The mathematical analysis of a similar system is given by Krasnogorskaya and Sedunov[6].

The teflon-insulated wire was removed from the system and the charges of the water drops were measured with the induction cage. The polonium cage was retained in order to neutralize charge on the drops. The drops were simultaneously started by a solenoid and passed through the polonium cage, an electric field, and then separated with one drop passing through the induction cage. With this system, the polonium cage would neutralize the charge on drops, the electric field would cause any transfer of charge between the surfaces of the drops, and the induction cage would measure if any charge was transferred and its magnitude.

This arrangement also proved to be very difficult to control. There was no way to get an accurate measurement of the distance between the surfaces of the drops to insure the proper condition of charge transfer, and for such small separations it was very difficult to separate the two drops for individual measurements of the charge. Also, if a charge transfer took place it could have been less than the lower measuring limit of the induction cage. No measurements gave any indication that charge was ever transferred.

It was decided that Sartor's approach with two glass fibers should be tried in order to verify his original observations. In this approach water droplets were placed on the tips of two glass fibers which were mechanically moved together in the presence of an electric field. This experiment eliminated the problem of timing the drops and allowed control of the initial spacing between the water surfaces.

The experimental setup is shown in Figure 6, and the following procedure was used. The drops were formed on the tip of a glass tubing and transferred to the glass fibers in the box. The initial horizontal separation between the drop surfaces was set with a micro-manipulater which supported the fibers from the top of the box. The electric field was increased in small increments and the water was observed through a ten power microscope. Normally the drops moved closer together with each increase in the electric field and, depending on the initial separation, the drops would coalesce for some particular field strength. Occasionally, the drops moved towards the plates or away from each other indicating the drops were charged. This charge was due to the surface leakage of the glass fibers and this was found to be very difficult to control. Various solutions of etch and cleaning agents were tried with only fair results. To prevent the water drop from creeping up the glass and causing a very deformed sphere, a thin coat of teflon was put on the surface of the glass. This coating was found to help in reducing the surface leakage.

The method of detecting the transfer of charge was visual; i.e., if a charge transfer occurred the drops should snap back to their original position since the transfer of charge would be such as to nullify the force pulling the drops together. But, of the few hundred pairs of drops examined, not one transfer of charge was indicated. Various values of electric fields

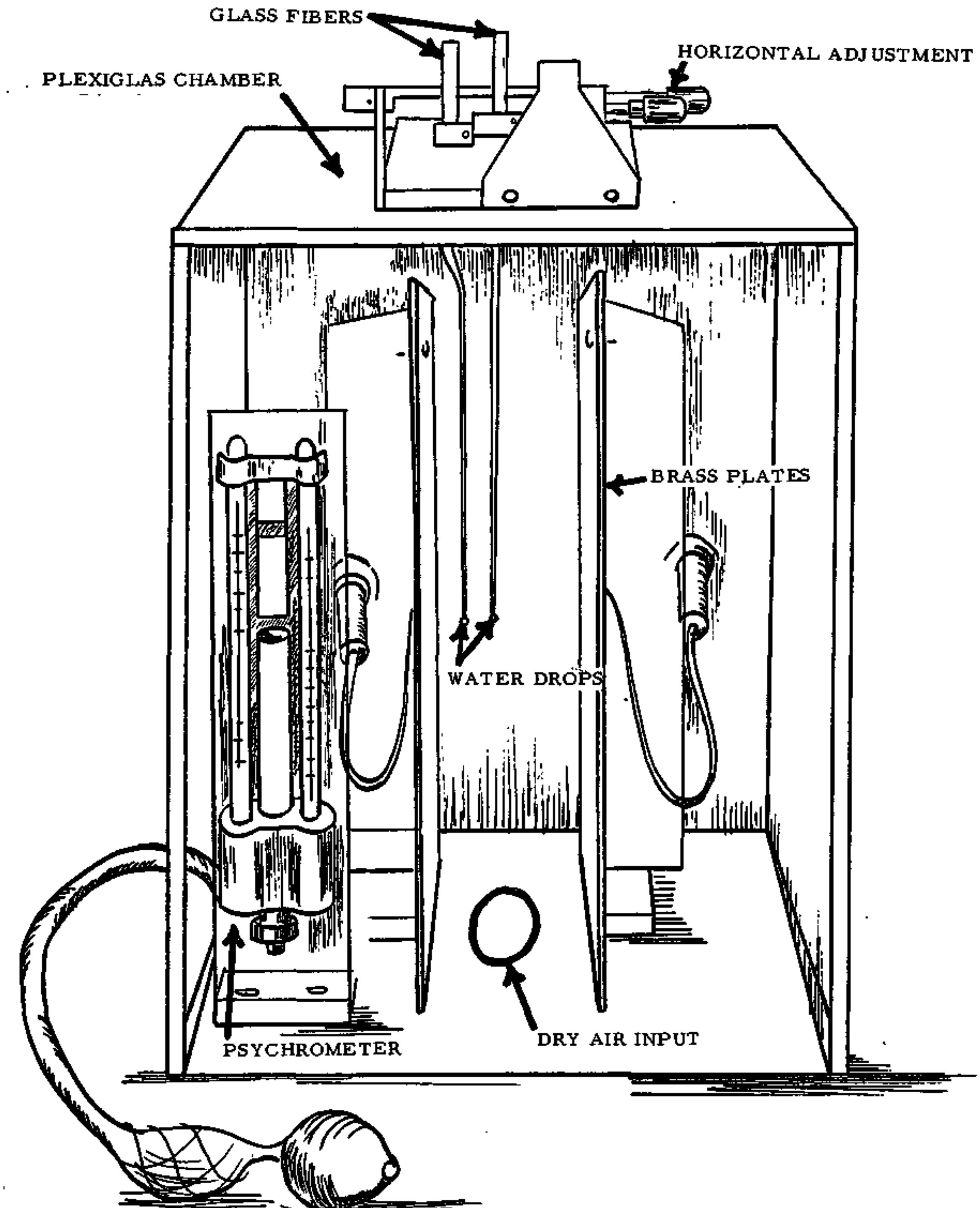

Figure 6

Apparatus to Observe the Behavior of Droplets in an Uniform Electric Field were pulsed in the vicinity of the drops. Again, the drops would either coalesce or gradually move toward the plates implying that they were becoming charged.

The water drops were then replaced with two steel balls mounted at the end of similar glass fibers. It was felt that by using rigid metallic spheres that the surface separation could be readily controlled. This allowed visual observation of the region between the surfaces. However, it was found that the fibers would bend slightly either together or apart, and that the spheres did not exhibit any relaxation motion or visual flashes.

The above experiments were conducted at about 20 C and 50 percent relative humidity. It is felt that the humidity should play an important part in any such phenomenon; therefore, a sealed system has been constructed in which the humidity can be controlled and measured in order to complete a full investigation of this possible charge-transferring mechanism.

C. Water Droplet Collision Experiment.

In order to obtain information about the phenomena of the collision and coalescence between two liquid surfaces, high speed photographs using a Fastax camera have been taken. One of the most obvious problems involved in gathering this type of information was in controlling the collision so that it will take place at a predicted point in space in order to focus a camera to photograph it.

The approach used was to form a charged beam of droplets from a hypodermic needle by using a ground plane and an electric field. Focusing electrodes were used in an attempt to focus these particles into a beam and to reduce their velocities. Two such beams were directed toward each other and photographs were taken. Several difficulties were encountered. In general, the droplets that came off the needle were quite random in size and were traveling in many directions. Under these conditions the focusing electrodes were never made to work very effectively in focusing these wide varieties of water particles. Therefore, since the field of focus for the magnifying lens used was small, only a few of the particles were in focus on the film taken and no collisions were photographed. Photographs were taken with both the same and the opposite charge on the particles of the two beams, but no appreciable difference was observed.

A more successful attempt resulted by forming a hemisphere of water on the top of a glass tube and letting a water droplet fall from above. By restraining one of the surfaces, it was possible to photograph the collisions from any angle.

An example of photographs taken at 9600 frames per second is shown in Figure 7. The glass tube was 6 mm in diameter and the falling droplet had a radius of about 1 mm. The droplet fell 20 centimeters and had a velocity of approximately 200 centimeters per second at impact. A slight flattening of the small droplet caused by the air flow can be observed. As the falling droplet collides with the hemisphere of water projected out of the lower tube, water from the hemisphere moves around and away from the surface of the falling droplet. This flow of water is caused by the momentum exchanged between the falling droplet and the hemisphere. However, the fact that the surface of the flow and the surface of the droplet do not coalesce can be explained by a thin film of air trapped between these two surfaces as suggested by Lang[4] .

For photographing free body collisions an apparatus generating uniform water droplets is now being constructed. It is made with a spring steel strip whose tip is pulled into the surface of a water reservoir and released by an electro-magnet activated by 60-cycle voltage. The spring steel strip is tuned to resonance, and small water droplets having radii on the order of 100 microns are pulled from the reservoir to follow a normal path of a free body. When adjusted properly, the drops are very uniform in size and are evenly spaced as they move away from the source. The operation is very sensitive to the water supply flow rate and to the adjustment of the tip with respect to the water surface. A second such apparatus will be built and aligned so that the beams will collide. If it is successful, then the collision of free liquid bodies can be photographed and such phenomena as coalescence and bounce-off can be studied.

D. Measurement of Terminal Velocity of Water Droplets

Gunn and Kinzer[7] have experimentally determined the terminal velocity, drag coefficient, and Reynolds number for distilled water droplets in stagnant air at 760 mm pressure, 50 percent relative humidity, and 20°C.

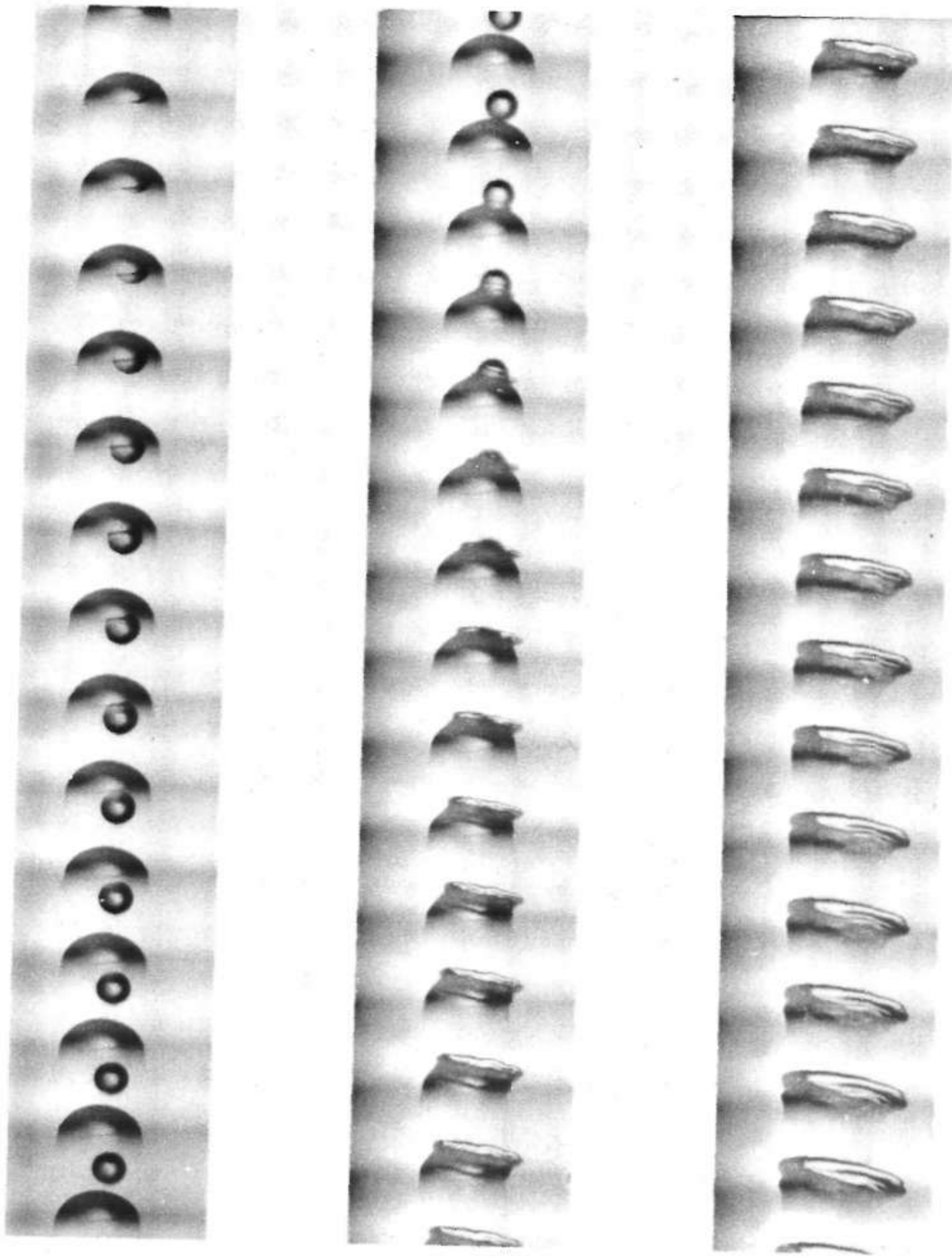

Figure 7

High Speed Photographs of Colliding Drops

The change in terminal velocity ,has not been determined for the conditions which are known to exist in clouds. The process of determining the effects of temperature, pressure, and water vapor content on the terminal velocity of water droplets is now underway. These experimental results will then be used in calculation of collision efficiency and to check against the theoretical expression derived by Davies[8]. The theoretical determination of the drag coefficient, Cd, Reynolds number, Re, and terminal velocity, V_t, will be achieved by first calculating the factor $C_d\ R_e^2$ which depends on the droplet radius, density of fluid and droplet, viscosity of fluid, and acceleration of gravity; equation given by Davies will then be applied.

, The principal sections of the apparatus as shown in Figure 8 are the drop tower; and the "control, system for humidity, temperature, and pressure.

The pressure control system has been constructed. The drying tower is 4 feet long and 6' inches in diameter. The tower will be filled with No. 2 mesh $CaSO_4$ which has an impedance to air flow equal to that of a 2-inch pipe. The fan chamber contains a fan and motor to circulate the air through the system. A vacuum pump reduces the pressure below the desired value in the drop tower; then the pressure will be raised to the desired value in the drop tower with the air stored in the storage tank. The humidity will be raised in the storage tank by passing the air over the heated water reservoir. However, the humidity will be sensed at the site of the experiment with an electric sensor in the drop tower.

The drop tower, which is 5 inches in diameter and 8 feet long, is nearly complete. Equipment such as induction rings to measure the terminal velocity is under construction. Prototype models have been tested and found to operate satisfactorily. The optical equipment used to measure the droplet size has yet to be installed. A photoelectric method described by Mikirov[9] will be used to measure the droplet size. The intensity of the light scattered by the droplets depends on the diameter. Since the scattered light is focused on the cathode of a phototube, the output voltage impulse obtained will be proportional to the size of the droplets.

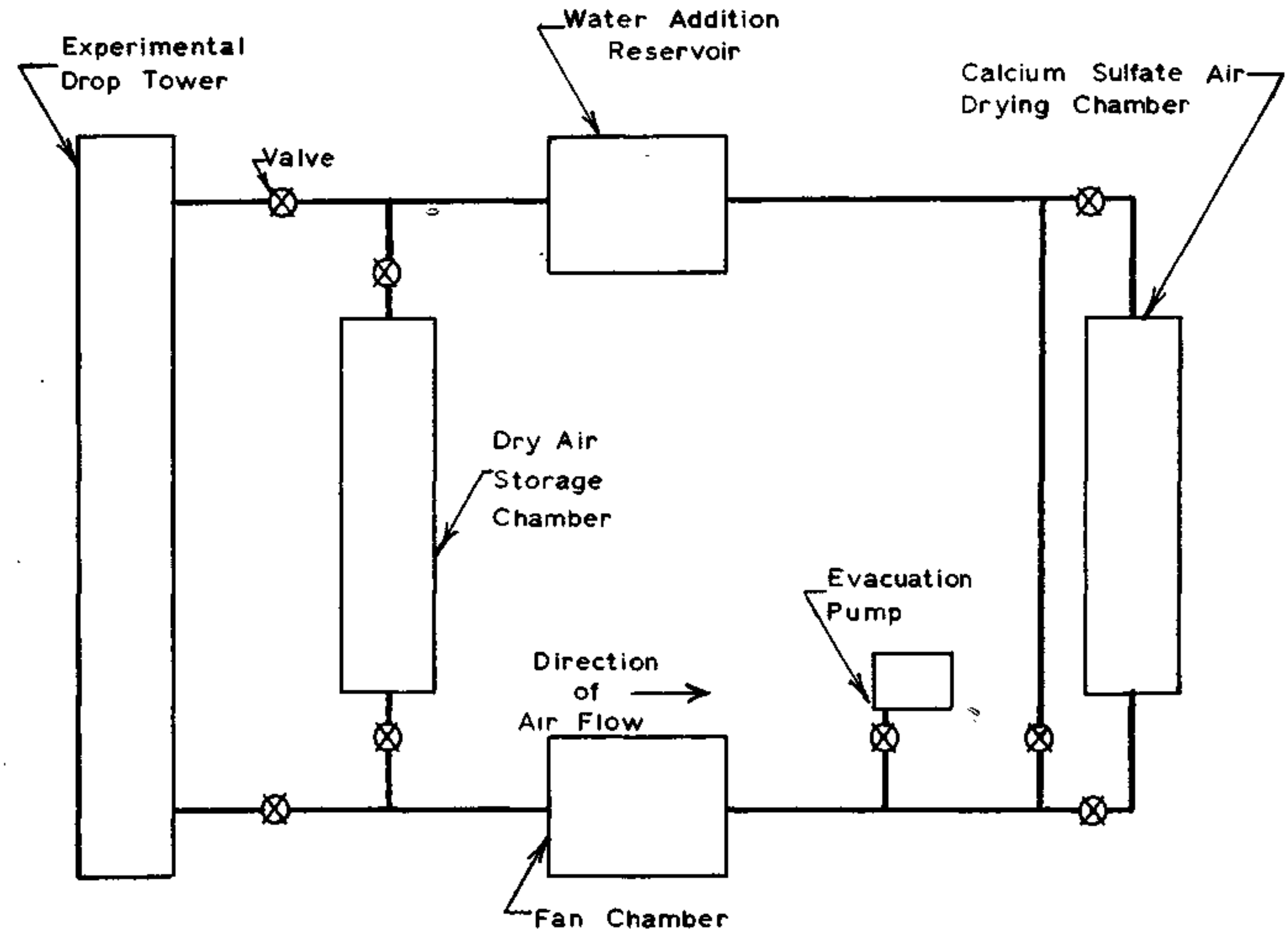

Figure 8

Terminal Velocity Apparatus for Humidity, Temperature
and Pressure Control

E. Summary

The drop chamber described in the report was designed to measure the thickness and shape of the air film trapped between two water droplets. The results indicate that a flow of dry nitrogen between the droplet surfaces prevents the two droplets from coalescing. Thus with the flow of nitrogen the droplets could be "rammed" together without coalescing. However, if the droplets were forced together without the nitrogen flow, that is, in dry stagnant air, the two droplets coalesced immediately. The main problem encountered in measuring the air-film thickness was the difficulty in being able to adjust the surface separation on the order of half wavelength of visible light. The fine adjustment used to bring the droplet surfaces into close proximity was too coarse and a micro-fine adjustment is now under construction.

In a number of experiments attempts were made to detect the charge transfer between two droplets approaching each, other in the presence of a uniform electric field. The experiments were carried out under both static and dynamic conditions. The static experiments were more favorable because the surface separation was controlled mechanically and two stationary droplets were easier to view. One major problem encountered was the surface leakage along the glass fiber rods used to support the droplets. This problem is still under investigation. Up to this time, charge transfer between two droplets has not been observed for various surface separations and electric field strengths.

Several experiments were initiated to examine the droplet collision process, of which the most successful was that of placing the two droplets on a vertical axis. The bottom droplet was a hemisphere formed at the end of a hollow tube and the top droplet was forced out of a second tube. Since the bottom droplet was stationary, the experiment did not simulate a free-body droplet collision occurring in a cloud. The high-speed photomicrographs taken of the collision indicate some. interesting results. The falling droplet penetrated about 0.75 drop diameters into the stationary droplet before coalescing. This could be explained by the presence of a thin film of air trapped between the two droplet surfaces. A device described in this report to simulate free-body collisions is still under construction.

Most of the instrumentation has been developed in order to adequately determine the effects of pressure, temperature and water vapor content on the terminal velocity of free falling water drops in stagnant air. The velocity measuring equipment includes methods for electrically charging the water droplets and using the resulting fields to sense the velocity at various points of the trajectory.

The preliminary research discussed above gives promise that many of the problems in this project will supply suitable topics for Ph.d. dissertations for the graduate students associated with the project.

3. THEORETICAL STUDIES

A. Determination of Collision Efficiencies

The calculations of Lindblad and Semonin[10] on collision efficiencies of pairs of droplets falling in a uniform electric field are being extended. Lindblad and Semonin assumed uncharged drops in a uniform field and considered only dipole interactions. In order to provide a more general treatment, both charged and uncharged droplets falling in a viscous medium in the presence of a uniform electric field of arbitrary intensity are being considered.

The differential equation for the motiona of the falling droplet includes electric, hydrodynamic, gravitational and xnertial forces. The equation of motion for a single drop may be written as

$$\frac{m d\vec{v}}{dt} \doteq -6\pi\mu a D \cdot (\vec{u} - \vec{V}) + m\vec{g} + \vec{F}_e$$

where the first term on the right is approximately the force due to hydrodynamic effects, the second term is the gravitational force and the third is the force due to electric fields acting on the droplet. The hydrodynamic term includes, through $\vec{V}$, the presence of the second drop and its effect on the first. This extension of the hydrodynamic treatment of the two-body problem for large Reynolds numbers has-been solved by Shafrir[11]. The term $\vec{F}_e$ also contains drop-drop interaction forces as well as the forces on the drop due to the externally applied electric field. Davis[12] has given a general solution for the forces acting on two conducting, rigid spheres in a uniform electric field. However, we are only concerned with the one-body problem with the more sophisticated electrostatics. In the one-body problem the fluid, containing the droplet, flows around the larger stationary drop. Collision efficiencies are being determined for: (1) uncharged cloud droplets falling in a field-free space and then in an electric field, and (2) charged cloud droplets falling in a field-free space and then in an electric field.

Laboratory studies by Sartor[5] show that at the point of nearest approach charge is frequently transferred between the droplets because of the strong fields between the surfaces. If this phenomenon does occur, it may have a considerable influence on the collision efficiency and droplet trajectory. Thus, the effects of charge transfer on collision efficiency and trajectory will be determined when the cloud droplets are in close proximity.

In order to solve the equations of motion of the two droplets, numerical methods must be used. For this purpose, the equations are being programmed for solution on an IBM 7090 computer. The method selected for the program is from Adams[13] as modified by Nordsieck[14]. Because the original work by Nordsieck was for a fixed point machine which is no longer in use, considerable modification of the program is necessary for use on a floating point machine (IBM 7090).

Briefly, the computation involves the application of a Taylor series expansion and retention of terms up to fifth order in the expansion. Nordsieck's application of Rutihauser's[15] stability criteria modifies the coefficients of the remainder terms of the Taylor series to produce a system of equations with limited memory of disturbances. This criterion is applied to provide optimum stability with greatest accuracy. The resulting stable equations allow a system of automatic interval changing to be employed which improves the accuracy of the solution in regions of rapid change.

Solutions obtained from the computer will be presented both as plots of drop trajectories and as a tabular compilation of collision efficiencies.

B. Electrical Effects on Droplet Surface

The effects of charge and electric fields are known to enhance the coalescence of water droplets. It has been demonstrated (1) that two colliding droplets do not always coalesce, because the thin film of air between the two droplets acts as a cushion. However, it has also been shown that in a strong electric field the two colliding droplets always coalesce. This presumably occurs because the electrical force is sufficient to overcome or "squeeze out" the thin film of air. To study the above problem and the effects of strong fields on droplet surfaces, calculation of the electrical force and fields between two droplets is required. The problem of two conducting rigid spheres has been solved by Davis[12] and programmed on a 7090 IBM computer by our staff. At present, the following results are available for two uncharged droplets, 30 and 5 microns in radius, in a uniform electric field directed along the line of their centers:

S = 5.0	microns	$E = 4.5E_o$
S = 0.5	microns	$E = 21.3E_o$
S = 0.05	microns	$E = 139.2E_o$
S = 0.005	microns	$E = 1031.0E_o$
S = 0.0005	microns	$E = 7261.0E_o$

where S is the droplet surface separation, E is the local field between the droplet surfaces, and E_o is the applied, external field of 3600 volts per centimeter. The above results indicate the enhancement of the local field over the ambient field as the separation between the surfaces of the two droplets decreases.

To study the effects that the local fields have on the droplet surface for different surface separations, the electrical pressure (force per unit area on the droplet surface) on the droplet was compared with the surface tension pressure. Assuming the surface tension forces are counteracted by the electrical forces acting on the droplet surface, an approximation for the effective surface tension is

$$\frac{2\gamma_E}{r} = \frac{2\gamma}{r} - \frac{E^2}{8\pi}$$

where $2\gamma_E/r$ is the effective surface tension pressure, $2\gamma/r$ is the surface tension pressure of a droplet, and E^2/π is the electrical pressure in c.g.s. units. Thus, from the above equation, the electrical forces overcome the surface tension forces when

$$E = (16\pi\gamma/r)^{1/2}$$

For the 30-micron and 5-micron droplets the effective surface tension is zero when the local electric fields are 330,000 and 872,000 volts per centimeter, respectively. The effective surface tensions for various surface separations are listed below

30-micron droplet:

$E = 4.5 E_o$	$S = 5.0$ microns	γ_E	$= 71.83$ dynes/cm
$E = 21.3 E_o$	$S = 0.5$ microns	γ_E	$= 68.25$ dynes/cm
$E = 1392 E_o$	$S = 0.05$ microns	γ_E	$= -95$ dynes/cm

5-micron droplet:

$E = 5.4 E_o$	$S = 5.0$ microns	γ_E	$= 71.97$ dynes/cm
$E = 21.3 E_o$	$S = 0.5$ microns	γ_E	$= 71.37$ dynes/cm
$E = 139.2 E_o$	$S = 0.05$ macrons	γ_E	$= 44.11$ dynes/cm
$E = 1031.0 E_o$	$S = 0.005$ microns	γ_E	$= -1458$ dynes/cm

The change in sign of γ_E from plus to minus indicates the surface tension forces have been overcome by the electrical forces for a separation between 0.5 and 0.05 for the 30-micron droplet and between 0.05 and 0.005 for the 5-micron droplet. Since the electric field is not uniform over the drop surface, deformation of the liquid drop will result because of the uneven surface stresses. The above analysis is only true for rigid spheres, therefore, for liquid drops in large fields the above equation for the effective surface tension is only a first order approximation.

C. Summary

The theoretical calculation of collision efficiencies for the three special cases mentioned in the report is continuing. Results will soon be available for uncharged cloud droplets falling in field-free space and in an electric field. The electrical force" for charged cloud droplets and the effects of charge transfer are to be programmed. With the more sophisticated electrostatic solution, collision efficiencies should be on the order of 10 to 30 percent higher than those calculted by Lindblad and Semonin.

The preliminary calculations on the effects of electric fields on droplet surfaces indicate that when the two droplets are in close proximity the surface tension forces can be overcome by the electrical forces acting on the surface. This occurs in the region where the field lines are concentrated. The indication of surface instability is an indication that deformation of the droplet surfaces is possible in strong electric fields.

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